Determine the abelian groups of order 900.

Let \( p \) be the smallest prime dividing the order of a finite group \( G \), and suppose \( G \) has a subgroup \( H \) of index \( p \). Show that \( H \) is normal.

Show that the ideal \( I \) in \( \mathbb{Z}[x] \) generated by 11 and \( x^2 + 1 \) is maximal.

Let \( S, T \) be diagonalizable operators on a finite-dimensional complex vector space \( V \). Suppose that \( ST = TS \). Show that there is a basis for \( V \) consisting of simultaneous eigenvectors for all \( S, T \).

Show that \( x^5 + y^7 + z^{11} \) is irreducible in \( \mathbb{C}[x, y, z] \).

Exhibit a finite field with 32 elements.

Let \( T \) be a diagonalizable operator on a finite-dimensional complex vector space \( V \). Given an eigenvalue \( \lambda \) of \( T \) on \( V \), show that there is a polynomial \( P \in \mathbb{C}[x] \) such that \( P(T) \) is the projector to the \( \lambda \)-eigenspace.

Explicitly determine all fields between \( \mathbb{Q} \) and \( \mathbb{Q}(\zeta) \), where \( \zeta \) is a primitive 12th root of unity.