Credit Implied Volatility

Team 1:
Ameya Phadke
Ali Nabizadeh
Heng Xu
Yifan Xu
Tim Berend

Mentor: Chris Bemis
Merton Model

- In the Merton Model, the firm’s value is equal to $V_t = E_t + D_t$
  - $E_t = \text{Market Cap at time } t$
  - $D_t = \text{Zero coupon debt, face value } K, \text{ maturity } T$

- Key Concepts in Merton model:
  - The value of the firm is the sum of its equity and debt
  - Equity can be interpreted as a call option
  - Debt can be interpreted as a short put option, and long cash

\[
E_T = \max(V_T - K, 0)
\]

\[
D_T = \begin{cases} 
V_T, & V_T < K \\
K, & V_T \geq K,
\end{cases} = K - \max(K - V_T, 0)
\]
Credit Spread Formula

- $V_0 = E_0 + D_0$
- Using Black Scholes...

\[ E = V_0 \Phi(d_1) - Ke^{-rt}\Phi(d_2) \]

\[ D = V_0 \Phi(-d_1) + Ke^{-rt}\Phi(d_2) \]

- After some computation... (y - r is the credit spread)

\[ y - r = -\frac{1}{T} \log\left(\frac{1}{L} [\mathcal{N}(-d_1) + L\mathcal{N}(d_2)]\right) \]

\[ L = \frac{Ke^{-rt}}{V_0} \]
Finding $V_0$ and $\sigma_v$

- These two non linear equations are used to find $V_0$ and $\sigma_v$ simultaneously:

\[ E = V_0 \Phi(d_1) - Ke^{-rt}\Phi(d_2) \]

\[ V_0\sigma_V \Phi(d_1) = E\sigma_E \]

- Using MatLab or Python, we find unique solutions $V_0$ and $\sigma_v$

- This process requires starting values of $E, r, T, K, \sigma_E$

\[ y - r = -\frac{1}{T} \log\left(\frac{1}{L} \left[ \mathcal{N}(-d_1) + L\mathcal{N}(d_2) \right] \right) \]
Credit Spreads Revisited

- Using our newly found $V_0$ and the credit spread formula, we are able to back out a credit implied volatility (CIV).
  - This method is performed exactly the same way as using options prices and Black-Scholes to back out an implied volatility of the stock.

\[
y - r = - \frac{1}{T} \log\left( \frac{1}{L} \left[ \mathcal{N}(-d_1) + L \mathcal{N}(d_2) \right] \right)
\]

- How does Credit implied vol compare to $\sigma_v$?
We know many of our assumptions are not realistic.

Can we get a better, more realistic fit?
Borland Model

- **Motivation**
  - Uses a non-Gaussian Asset Model
  - Incorporates skew(ᶓ) and kurtosis(q)
  - Could lead to a single volatility for the firm

- **Non-Gaussian, statistical feedback process** with $\alpha$ and $q$ instead of the standard lognormal process:

\[
dV = \mu V \, dt + \sigma V_0^{1-\alpha} V^\alpha \, d\Omega \\
d\Omega = P^{(1-q)/2} (\Omega) \, d\omega
\]
Returns Graph / Histogram (Borland Method)
Skew and Kurtosis

Graph of Skewness for values of $\alpha$

Graph of Kurtosis for values of $q$
Borland Model Structure

- The Merton style argument will still apply to relate credit spreads in the Borland Model
  - Now we have $\sigma$, $\alpha$ and $q$ in the model
- Equity is still viewed as a call, but the equation is a bit different

\[
E_0 = V_0 \int_{d_1}^{d_2} (1 + (1 - \alpha)x(\hat{T})) \frac{1}{1-\alpha} P_q(\Omega_{\hat{T}})d\Omega_{\hat{T}} - e^{-rT}D \int_{d_1}^{d_2} P_q(\Omega_{\hat{T}})d\Omega_{\hat{T}}
\]

\[
E_0 \sigma_E - \frac{\partial E}{\partial V} V_0 \sigma_V = 0
\]

- However, the math is significantly more challenging than the Merton model
Solution Strategy

- Because we are brave mathematicians, our approach will be the following:
  - Using the equations from the previous slides, we will obtain $V_0$ and $\sigma_v$ in a similar fashion to the Merton model
  - After $V_0$ is found, use the following formulas to derive our credit spread:

\[
D_0 = V_0 - E_0
\]

\[
y - r = -\frac{1}{T} \log\left(\frac{D_0}{D_e^{-rT}}\right)
\]
Numerical Complications

- Solving for $V_0$ and $\sigma_v$ is quite unstable.
- Specifically, solving for $V_0$ and $\sigma_v$ is sensitive to $\alpha$ and $q$ parameters.
- Values of $\alpha$ and $q$ will be restricted to a range where credit spread is not a complex number.
  - $\alpha$ is between .3 and .99
  - $Q$ is between 1.23 and 1.7
- Using this range, credit spreads can be produced for comparison to the market CDS spreads.
Spread Formula Optimization

- Our goal is to obtain theoretical spreads and compare them to spreads quoted in the CDS market.
- Recall Credit Implied Vol displayed a term structure.
  - We will apply this phenomena to compare theoretical and market spreads.
- But how?
Spread Formula Optimization

- **Steps:**
  - Define an objective function →
  - $S_{q,\alpha}$ is the spread derived from formulas
  - $S(t)$ is the spread from the market

$$f(\alpha, q) = \sum_{t=1}^{n} (S(t) - S_{q,\alpha}(t))^2$$

- Minimize this function. Doing this will find the function of $\alpha$ and $q$ that fits the term structure curve of CDS spreads closest (smallest sum of residuals)
- We then obtain the best approximations $\alpha$ and $q$
Homestretch

- After we find the optimal $\alpha$ and $q$ from our best fitting curve, how does this help us?
- We repeat our previous process, solving for $V_0$ and $\sigma_v$ simultaneously.
- The difference this time is that we fix $\alpha$ and $q$ as the values obtained from the best fitting curve.
- This should yield a unique $V_0$ and $\sigma_v$.
  - $\sigma_v$ is the volatility of the firm we are looking for.
Results

- Using the intervals for $\alpha$ and $q$, we were able to fit a curve relating the derived spreads with those found in the market for AAPL CDS.
  - $\alpha = 0.3325$
  - $q = 1.5157$
Results

- Finally, with optimal $\alpha$ and $q$, we can get a single volatility
- In this case, for AAPL, we get a $\sigma_v$ of .2051 or 20.51%
- This represents the single volatility that describes the assets of the firm

Next Steps

- Use Monte-Carlo simulations to price call options with Borland Model
- Use that call option equation for the equity