TEAM 2 FINAL REPORT

Constructing a no-arbitrage volatility surface in liquid and illiquid commodity markets

https://sites.google.com/site/fmmodeling11/
SVI Model

Parameterization:

\[ \text{var}(k; a, b, \sigma, \rho, m) = a + b\{\rho(k - m) + \sqrt{(k - m)^2 + \sigma^2}\} \]

- \(a\) gives the overall level of variance
- \(b\) gives the angle between the left and right asymptotes
- \(\sigma\) determines how smooth the vertex is
- \(\rho\) determines the orientation of the graph
- \(m\) translates the graph
In Commodity market

- Commodity Data
  - Corn (liquid)
  - Live Cattle (somewhere in between)
  - Milk (illiquid)
Find Implied Volatility

- Bisection not Newton’s method
- Only out-of-money data

![Graphs of Corn and Milk](image_url)
Fitting the SVI Model

- Use the method of least square
  - Minimize the objective function:
    \[ \sum (\sigma_{SVI} - \sigma_{Mkt})^2 \]
    where
    \[ \sigma_{SVI} = \text{var}(k; a, b, \sigma, \rho, m) = a + b\left\{ \rho(k - m) + \sqrt{(k - m)^2 + \sigma^2} \right\} \]
  - No constraints imposed
Fitting the SVI Model

Use the method of least square to minimize the objective function:

\[ \sum_{k=1}^{n} \left( \sigma_{SVI}(k, a, b, m) - \sigma_{SVI, Mkt}(k) \right)^2 \]

where no constraints imposed.
Fitting the SVI Model

Use the method of least square

Minimize the objective function: where

No constraints imposed

Fitting the SVI Model

Sep11 Corn
Replicating digital options

- Bull call spread to replicate digital call
Replicating digital options

- Bull call spread to replicate digital call
- Bear put spread to reproduce digital put
Replicating digital options

- Equation for pricing digital call
  \[-\frac{\partial C}{\partial K} - \text{Vega} \cdot \frac{\sigma_1 - \sigma_2}{K_1 - K_2}\]
Replicating digital options

- Equation for pricing digital call
  \[ -\frac{\partial C}{\partial K} - \text{Vega} \cdot \frac{\sigma_1 - \sigma_2}{K_1 - K_2} \]

- Equation for pricing digital put
  \[ \frac{\partial P}{\partial K} + \text{Vega} \cdot \frac{\sigma_1 - \sigma_2}{K_1 - K_2} \]
Replicating digital options

- Equation for pricing digital call
  \[ -\frac{\partial C}{\partial K} - \text{Vega} \cdot \frac{\sigma_1 - \sigma_2}{K_1 - K_2} \]

- Equation for pricing digital put
  \[ \frac{\partial P}{\partial K} + \text{Vega} \cdot \frac{\sigma_1 - \sigma_2}{K_1 - K_2} \]

- Relation with Black-Scholes
  - Call: \[ -D_{BS} - \text{Vega} \cdot \frac{\sigma_1 - \sigma_2}{K_1 - K_2} \]
  - Put: \[ D_{BS} + \text{Vega} \cdot \frac{\sigma_1 - \sigma_2}{K_1 - K_2} \]
Formula difference

Method #1 Black Scholes

\[ C = e^{-rT} \Phi(d_2). \]

Method #2 Replication

\[ C = \lim_{\epsilon \to 0} \frac{C_v(K - \epsilon) - C_v(K)}{\epsilon} = -\frac{dC_v}{dK} \]
\[ = -\frac{dC_v(K, \sigma(K))}{dK} - \frac{\partial C_v}{\partial K} \frac{\partial \sigma}{\partial K} \]
\[ - \frac{\partial C_v}{\partial K} = - \frac{\partial (S \Phi(d_1) - Ke^{-rT} \Phi(d_2))}{\partial K} = e^{-rT} \Phi(d_2) = C_{noskew} \]

Option Vega

Slope of volatility skew
Comparison

CZ0 Volatility Skew

ATM Corn Price Difference

![CZ0 Volatility Skew Graph](image1)

![ATM Corn Price Difference Graph](image2)

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Comparison

**CZ0 Volatility Skew**

**OTM Corn Price Difference**
Comparison

CZ0 Volatility Skew

Deep OTM Corn Price Difference

![Graph of CZ0 Volatility Skew](image)

![Graph of Deep OTM Corn Price Difference](image)
Comparison

**LCM1 Volatility Skew**

**ATM LCM1 Price Difference**

![Graph showing LCM1 Volatility Skew]

![Graph showing ATM LCM1 Price Difference]

**Team 2 Final Report**
Comparison

DAJ1 Volatility Skew

OTM LCM1 Price Difference

-0.3  -0.25  -0.2  -0.15  -0.1  -0.05  0  0.05  0.1  0.15
0 0.05 0.1 0.15 0.175 0.17 0.165 0.16 0.155 0.15 0.145 0.14 0.135

Vol (%) vs log(K/S)

10^-3  10^-2  10^-1  10^0  10^1  10^2  10^3
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14

Digital option price vs Tick Size

Moderately out of the money (Live Cattle)
Comparison

DAJ1 Volatility Skew

Deep OTM LCM1 Price Difference
Comparison

DAJ1 Volatility Skew

DAJ1 ATM Price Difference

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Comparison

DAJ1 Volatility Skew

DAJ1 OTM Price Difference
In equity market

Where we have volatility surface
Fitting SVI curve

- Unconstrained Optimization
  - MATLAB – fminsearch

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<th></th>
<th>a</th>
<th>B</th>
<th>σ</th>
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<tr>
<td></td>
<td>0.23</td>
<td>4.01</td>
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<tr>
<td>ρ</td>
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<td>m</td>
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Fitting SVI curve

- **Constrained Optimization**
  - MATLAB – fmincon
  - \( a > 0, b > 0, |\rho| < 1 \)

<table>
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<th>a</th>
<th>b</th>
<th>( \sigma )</th>
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<td>0</td>
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<tr>
<td>( \rho )</td>
<td>-1</td>
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## Fitting SVI curve

- **Constrained Optimization**
  - **MATLAB – fmincon**
  - $|\rho| < 1$

### Table

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<tr>
<td>ρ</td>
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<td>-0.54</td>
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### Graph

![Graph of SVI curve fitting](image)

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Implied Volatility Skews for Equities
Implied Volatility Skews for Equities

![Graph of implied volatility skews for April 11 and June 11.](image)
Implied Volatility Skews for Equities

Sep 11

Dec 11

Vol (%) vs log(\$S)
SVI Volatility Surface

- Forward Volatility
  - Thanks Jason’s team
Summary of what we have done

- Find implied volatility for commodities
- Parameterize SVI variance curves
- Price digital options with volatility skew
- Build SVI implied volatility skews for SPX
- Get the volatility surface from forward volatility
- Price SPX digital
- Compare between Black-Scholes pricing evaluation and replication pricing mechanism.
Constructing a no-arbitrage volatility surface in liquid and illiquid commodity markets
QUESTIONS
& COMMENTS