Adjusting the Black-Scholes Framework to Account for Volatility Skew

Chris Prouty : Cargill
• To understand the problem, we must first define a few things...
• Hopefully everyone is familiar with the Black-Scholes model
  • Recall that there are a number of assumptions underlying Black-Scholes
  • Constant interest rates, constant volatility, the underlying follows a Brownian motion, etc...
• For a given underlying, there are a couple definitions of volatility
  • Implied volatility
  • Realized volatility
• Realized volatility can be defined as the annualized standard deviation of the log returns of the security
• Implied volatility can be defined as the volatility used to price an option of a specific tenor and a specific strike, referencing a specific underlying.
• Under the Black-Scholes framework, for a given tenor, the implied volatility should be the same at all strikes.
  • Actual options traded in the marketplace were formerly priced in this manner.
  • A single watershed event changed the structure of implied volatility.
  • Traders realized that perhaps returns are not always normally distributed!
• To account for a painful lesson regarding flaws of the Black-Scholes model, a heuristic was adopted by traders.
The volatility skew/smile was born
- Although mitigating one type of risk, this artifact of option markets introduces a new kind of risk to traders
  - Recall the Black-Scholes Greek “Vega”
  - Vega measures how much an option price changes with respect to a change in implied volatility
- Given the axiom that the skew tends to move with the underlying, a risk is presented
- Consider the risk presented if a trader sells an at-the-money option assuming the skew shown in red
  - Vega risk is now presented contingent on a move in the underlying
• Our project will focus on adjusting the traditional delta value (and the corresponding delta hedge) to account for changes in the option value due to shifts in the volatility skew.
• We will begin with strict assumptions about the nature of the skew and attempt to explore the problem.
• After understanding the problem, we will generalize the problem and explore existing literature and industry approaches.
• Ultimately, we aim to design a framework that allows us to calculate a robust delta for an arbitrary volatility surface.