PROGRAM

(All the talks will be in Room 113, Vincent Hall)

Wednesday, June 23rd

09,00 coffee and donuts at the common room

09,45-10,45 **Gary Lieberman**
Anisotropic elliptic equations: regularity of solutions

11,00-12,00 **Hans-Christoph Grunau**
A decomposition method in higher order Sobolev spaces and some application

14,30-15,30 **James Serrin**
Qualitative behavior of singular elliptic equations with weights

15,30 coffee break

16,00-17,00 **Patrizia Pucci**
Existence of radial ground states for p-Laplacian elliptic equations with weights

Thursday, June 24th

09,00 coffee and donuts at the common room

09,45-10,45 **Guido Sweers**
Some questions on elliptic systems concerning domain shape

11,00-12,00 **Henghui Zou**
Existence for strongly coupled semilinear elliptic cooperative systems via a priori estimates

14,30-15,30 **Moxun Tang**
Uniqueness of positive radial solutions for $\Delta u - u + u^p = 0$ in annular domains

15,30 coffee break

16,00-17,00 **Biao Ou**
A Poincaré inequality on $\mathbf{R}^n$ and its application to potential fluid flows

Friday, June 25th

09,00 coffee and donuts at the common room

09,45-10,45 **Hans Weinberger**
Nonlinear p-homogeneous first order ODE systems and their linear perturbations

11,00-12,00 **Giovanni Leoni**
On optimal regularity of free boundary problems and a conjecture of De Giorgi

14,00-15,00 **Grozdena Todorova**
Asymptotic behavior and regularity for wave equations with nonlinear dissipative term in $\mathbf{R}^n$

15,00 coffee break

15,30-16,30 **Filippo Gazzola**
Semilinear parabolic equations with initial data at high energy level

ABSTRACTS

Semilinear parabolic equations with initial data at high energy level

Filippo Gazzola
Politecnico di Milano – Milano (Italy)

Let Ω be an open bounded domain of $\mathbf{R}^n$ $(n \geq 2)$ with smooth boundary $\partial\Omega$. Depending on suitable properties of the initial datum u_0, we are interested in existence of both finite time blow-up solutions and of solutions which exist globally in time of the following parabolic problem

$$\begin{cases} u_t - \Delta u = |u|^{p-1}u & \text{in } \Omega \times (0,T) \\ u(0) = u_0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \times (0,T) \end{cases}$$

where $u_0 \in H^1_0(\Omega)$, $T \in (0,\infty]$ and $1 < p < \frac{n+2}{n-2}$, understanding that $\frac{n+2}{n-2} = +\infty$ if $n = 2$.We determine the behavior of solutions having initial data u_0 with high energy, namely above the mountain pass level. We make use of comparison and smoothing principles as well as methods from critical point theory, from potential well theory and from semigroup theory. We obtain both global solutions and finite time blow-up solutions.

A decomposition method in higher order Sobolev spaces and some application

Hans-Christoph Grunau
Otto-von-Guericke-Universitaet – Magdeburg (Germany)

This talk concerns joint work with Filippo Gazzola, Enzo Mitidieri and Marco Squassina. Much of the successful development of second order elliptic equations in the past decades is based on the decomposition$u = u^+ - u^- \in W^{1,2}_0(\Omega)$. This decomposition is no longer admissible in higher order Sobolev spaces $W^{k,2}_0(\Omega)$ with $k \geq 2$. This is in my opinion an important reason for our relatively restricted knowledge of higher order nonlinear elliptic equations. In 1961, generalizing the projection theorem, Moreau gave a decomposition result with respect to pairs of dual cones $\mathcal{K}, \mathcal{K}^*$. He already emphasized that when taking $\mathcal{K} \subset W^{1,2}_0(\Omega)$ the cone of nonnegative functions, one gainsa decomposition $u = u_1 + u_2 \in W^{1,2}_0(\Omega)$ with $u_1 \in \mathcal{K}$, $u_2 \in \mathcal{K}^*$ being weakly subharmonic.By the maximum principle, one has $\mathcal{K}^* \subset -\mathcal{K}$.

In $W^{k,2}_0(\Omega)$ with $k \geq 2$, in some cases, this decomposition may replace the previous one into u^+ and u^-. By a restricted comparison result, at least in some domains or under suitable boundary conditions, we still have $\mathcal{K}^* \subset -\mathcal{K}$, even if $k \geq 2$. This method will be explained in detail. Among others, applications will be given to the nonlinear critical growth problem

$$\Delta^2 u = |u|^{8/(n-4)}u \qquad \text{in } \Omega$$

under various suitable boundary conditions.

On optimal regularity of free boundary problems and a conjecture of De Giorgi

Giovanni Leoni
Carnegie Mellon University – Pittsburgh (USA)

We present some results on the optimal regularity of solutions of one and two-phase free boundary problems and of free discontinuity problems. We are particularly interested in the analyticity of local minimizers of the Mumford-Shah functional and on a conjecture of De Giorgi.

Anisotropic elliptic equations: regularity of solutions

Gary Lieberman
Iowa State University – Ames (USA)

Our model equation is

$$\left(|u_x|^{m-2}u_x\right)_x + \left(|u_y|^{q-2}u_y\right)_y = 0$$

with unequal parameters m and q, both greater than 1. Such equations have received considerable interest recently and most results have only been proved if m and q are sufficiently close to each other. In this talk, we show that some surprising regularity results aretrue without any restrictions on the closeness of m and q. A key element is a suitable version of the Sobolev inequality due to Michael and Simon.

A Poincaré inequality on $\mathbf{R}^n$ and its application to potential fluid flows

Biao Ou
University of Toledo – Toledo (USA)

Consider a function $u(x)$ in the standard localized Sobolev space $W^{1,p}_{loc}(\mathbf{R}^n)$ where $n \geq 2$, $1 \leq p < n$. Suppose that the gradient of $u(x)$ is globally L^p integrable; i.e.,$\int_{\mathbf{R}^n} |\nabla u|^p dx$ is finite. We prove a Poincaré inequality for $u(x)$ over the entire space $\mathbf{R}^n$. Using this inequality, we prove that the function subtracting a certain constant is in the completion of $C^\infty_c(\mathbf{R}^n)$ with respect to the norm$||\phi|| = (\int_{\mathbf{R}^n} |\nabla \phi|^p dx)^{1/p}$ where $\phi \in C^\infty_c(\mathbf{R}^N)$. As a result, we come to know the best constant and the optimizing functions for the Poincaré inequality on $\mathbf{R}^n$.We then prove a similar inequality for functions whose higher order derivatives are L^p integrable on $\mathbf{R}^n$.Next we study functions whose gradients are L^p integrable on an exterior domain of $\mathbf{R}^n$ and apply the results to another proof of an existence theorem for irrotational and incompressible flowsaround a body in space. This is joint work with Guozhen Lu.

Existence of radial ground states for p-Laplacian elliptic equations with weights

Patrizia Pucci
Università di Perugia – Perugia (Italy)

Using the main qualitative properties established in a recent paper by Pucci, Garcia-Huidobro, Manasevich and Serrin, as well as a modified identity of the type of Ni, Pucci and Serrin, we prove existence of radial semi-classical ground states of p-Laplacian elliptic equations with weights, when the nonlinearity $f = f(u)$, $u \geq 0$, is subcritical at ∞ in a general way.

Qualitative behavior of singular elliptic equations with weights

James Serrin
University of Minnesota – Minneapolis (USA)

In many papers the question of uniqueness of radial ground states for the equation $\Delta u + f(u) = 0$ and for various related equations has been studied. It is remarkable that throughout this work (except in very special circumstances) nowhere are spatially dependent weight terms taken into consideration. Here we shall discuss this problem and in particular establish various qualitative properties of solutions for this purpose.

Some questions on elliptic systems concerning domain shape

Guido Sweers
TUDelft – Delft (Netherlands)

Fix $\Omega \subset \mathbf{R}^n$ and consider Brownian motion that starts at $x \in \Omega$ and that is killed when reaching the boundary or an infinitesimal small ball around $y \in \Omega$. If $\mathbf{E}_y^x(\tau_\Omega)$ denotes the expected lifetime for the paths connecting x to y, for which pair (x, y) is this quantity maximal? And if we fix $|\Omega|$ for which Ω is $\sup_{x,y\in\Omega} \mathbf{E}_y^x(\tau_\Omega)$ maximal or minimal? These questions can be rephrased in terms of elliptic systems. The answers are related with positivity preserving properties for systems of elliptic equations that are coupled in a noncooperative way.
Results are obtained in collaboration with B. Kawohl (Cologne) and with A. Dall'Acqua (Delft) and H.-Ch. Grunau (Magdeburg).

Uniqueness of positive radial solutions for $\Delta u - u + u^p = 0$ in annular domains

Moxun Tang
Michigan State University – East Lansing (USA)

In this talk I will present a new identity which leads to the proof of uniqueness of positive radial solutions to the semilinear elliptic equation

$$\Delta u - u + u^p = 0$$

subject to the Dirichlet boundary condition in annular domains. I will also demonstrate that the use of the same identity provides a much simpler, possible the simplest, proof for the uniqueness of positive solutions to the same problem in a finite ball, or in the whole space $\mathbf{R}^n$.

Asymptotic behavior and regularity for wave equations with nonlinear dissipative term in $\mathbf{R}^n$

Grozdena Todorova
University of Tennessee – Knoxville (USA)

The energy decay estimate for nonlinear dissipative wave equations is an important question and a key starting point for many problems. The best known till now result is a logarithmic decay of the energy for the pure power nonlinear dissipative equation in $\mathbf{R}^n$. In order to improve this decay we have found several new facts arising from the presence of the dissipative term. We use these facts to derive polynomial decay of the energy for nonlinear dissipative wave equations in $\mathbf{R}^n$.

Nonlinear p-homogeneous first order ODE systems and their linear perturbations

Hans Weinberger
Minnesota University – Minneapolis (USA)

Recent joint work with Hirokazu Ninomiya on the large time behavior of solutions of the system

$$\frac{d\mathbf{u}}{dt} = \mathbf{f}(\mathbf{u})$$

where $\mathbf{f}$ is a homogeneous function of degree $p > 1$ in the d-vector $\mathbf{u}$, and of solutions of the linearly perturbed system

$$\frac{d\mathbf{u}}{dt} = \mathbf{f}(\mathbf{u}) + \alpha A\mathbf{u}$$

where A is a matrix will be discussed.

Existence for strongly coupled semilinear elliptic cooperative systems via a priori estimates

Henghui Zou
University of Alabama – Birmingham (USA)

We establish a priori supremum estimates for non-negative solutions of strongly coupled semilinear cooperative elliptic systems via blow-up. With a priori estimates available, we then derive an existence result of positive solutions for such systems via the classical fixed point theorems.