Outline

1. Introduction

2. What do academic actuaries do? Examples of applied actuarial research from my files.
   i. Risk measures
   ii. The RSLN model
   iii. Hybrid pension plans
   iv. GMIBs

3. Working together for better risk management.

4. Onwards and upwards.
ACADEMIC/INDUSTRY RELATIONS

- Practitioners exasperated by academics
  - work that is too complex
  - irrelevant
- Academics exasperated by practitioners
  - Ignore academics’ work
  - complain about standard scientific method
  - cling to obsolete technology
Practitioners (some) want
  • Quick answers to short-term problems
  • Methods not models
  • Safe harbour strategies
  • Math-lite communications

Academics want
  • Peer-review publications and citation indices
  • Scientific approach and record
  • Financial support
EXAMPLE 1

RISK MEASURES
Quantile risk measures

VaR

CTE/TCE/WCE/ExpectedShortfall/TailVaR/CVaR

are (in most applications) all the same…
Notation.

(a) We shall call $\Omega$ the set of states of nature, and assume it is finite. Considering $\Omega$ as the set of outcomes of an experiment, we compute the final net worth of a position for each element of $\Omega$. It is a random variable denoted by $X$. Its negative part, $\max(-X, 0)$, is denoted by $X^-$ and the supremum of $X^-$ is denoted by $\|X^-\|$. The random variable identically equal to 1 is denoted by $\mathbf{1}$. The indicator function of state $\omega$ is denoted by $\mathbf{1}_{\{\omega\}}$.

(b) Let $\mathcal{G}$ be the set of all risks, that is the set of all real valued functions on $\Omega$. Since $\Omega$ is assumed to be finite, $\mathcal{G}$ can be identified with $\mathbb{R}^n$, where $n = \text{card}(\Omega)$. The cone of non-negative elements in $\mathcal{G}$ shall be denoted by $L_+$, its negative by $L_-$.

(c) We call $\mathcal{A}_{i,j}, j \in J_i$, a set of final net worths, expressed in currency $i$, which, in country $i$, are accepted by regulator/supervisor $j$.

(d) We shall denote $\mathcal{A}_i$ the intersection $\bigcap_{j \in J_i} \mathcal{A}_{i,j}$ and use the generic notation $\mathcal{A}$ in the listing of axioms below.
Axiom 2.1. The acceptance set $A$ contains $L_+$. 

Axiom 2.2. The acceptance set $A$ does not intersect the set $L_{--}$ where 

$$L_{--} = \{X \mid \text{for each } \omega \in \Omega, \ X(\omega) < 0\}.$$ 

Axiom 2.3. The acceptance set $A$ is convex.

A less natural requirement on the set of acceptable final net worths is stated in the next axiom.

Axiom 2.4. The acceptance set $A$ is a positively homogeneous cone.
Definition 5.1. Tail conditional expectation (or “TailVaR”): given a base probability measure $\mathbb{P}$ on $\Omega$, a total return $r$ on a reference instrument and a level $\alpha$, the tail conditional expectation is the measure of risk defined by

$$TCE_\alpha(X) = -\mathbb{E}_{\mathbb{P}}[X/r \mid X/r \leq -VaR_\alpha(X)].$$

Definition 5.2. Worst conditional expectation: given a base probability measure $\mathbb{P}$ on $\Omega$, a total return $r$ on a reference instrument and a level $\alpha$, the worst conditional expectation is the coherent measure of risk defined by

$$WCE_\alpha(X) = -\inf\{\mathbb{E}_{\mathbb{P}}[X/r \mid A] \mid \mathbb{P}[A] > \alpha\}.$$
The Conditional Tail Expectation with parameter $\alpha$ (CTE$_\alpha$) is defined as the expected value of a loss, given that the loss falls in the upper $(1 - \alpha)$ tail of the distribution, $0 \leq \alpha < 1$.

Also,

$$CTE_\alpha(X) = \hat{\Phi}_0 \ g(S(x))dx, \quad g(t) = \begin{cases} \frac{t}{1-\alpha} & 0 < t \leq 1 - \alpha \\ \frac{1}{1-\alpha} & 1 - \alpha < t < 1 \end{cases}$$
HOW DID THESE PAPERS FARE?

Artzner et al:
- Tremendously successful paper; > 3000 citations
- Most important risk management paper in 20 yrs
- Highly rigorous; opened up research avenues
- But Solvency II creators use VaR.

Wirch & Hardy:
- Decent citation count, but…
- CTE is ubiquitous in North American insurance risk management.
Which is the success and which is the failure?
EXAMPLE 2

REGIME SWITCHING LOG NORMAL MODEL FOR LONG TERM STOCK RETURNS (RSLN)
RSLN MODEL

- Paper published in NAAJ (2001)
  - fatter tailed model for P-measure (cf GBM)
  - Tractable, easy to simulate, empirically consistent with major indices
  - Relatively easy to expand to multiple indices
  - Suggested for modelling hedged or un-hedged embedded option costs
Regime 1: $\rho_t = 1$
$\logN(\mu_1, \sigma_1^2)$

$P_{12}$

Regime 2: $\rho_t = 2$
$\logN(\mu_2, \sigma_2^2)$

$P_{21}$
RSLN-2

- Widespread implementation in industry
  - In major software packages
  - Basis for calibration – benchmark tests
  - We anticipated the black swans…. 
## RSLN-2 CALIBRATION TEST

<table>
<thead>
<tr>
<th>Accumulation period</th>
<th>2.5(^{th}) percentile</th>
<th>5(^{th}) percentile</th>
<th>10(^{th}) percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>One year</td>
<td>0.76</td>
<td>0.82</td>
<td>0.90</td>
</tr>
<tr>
<td>Five years</td>
<td>0.75</td>
<td>0.85</td>
<td>1.05</td>
</tr>
<tr>
<td>Ten years</td>
<td>0.85</td>
<td>1.05</td>
<td>1.35</td>
</tr>
</tbody>
</table>

**TABLE 1**
RSLN-2

- RSLN less popular amongst academics
  - critiqued for discrete time approach
  - “inelegant” cf Levy processes

SUCCESS OR FAILURE?
 HYBRID PENSION PLANS

• Defined Benefit plans – volatile costs
• Defined Contribution plans – volatile benefits
• Hybrid plans combine DB & DC
• Floor Offset = DB Underpin offers
  \[
  \text{max} (\text{DC Pension, DB Pension})
  \]
• The idea is that the DB payout is rare.
THE DB UNDERPIN

- $B_t = \alpha n Z_t$  where
  - $\alpha$ = accrual rate, $n$ = service at $t$,
  - $Z_t$ = final average salary at $t$

- $DB_T = B_T a(T) = \text{value of DB pension at } T$

- $DC_T = \text{Value of DC funds at } T$
The pension cost is

\[ \max(DB_T, DC_T) \]

\[ = DC_T + \max(DB_T - DC_T, 0) \]

• Compare with exchange option payout

\[ \max(S^1_T, S^2_T) = S^1_T + \max(S^2_T - S^1_T, 0) \]
ACTUARIAL FUNDING

- **Accruals funding**
  - Liability recognized when service is completed
    - Past service benefit = accrued benefit
  - Contributions (NC) fund the expected increase in liability between valuations, using
    - projected future salary (PUC) or
    - current salary (TUC)
FUNDING

• Value of guarantee using Black-Scholes framework (and some heroic assumptions)

\[ H(t) = E_t^Q \left( e^{-r(T-t)} \left( a_t Z_t a(t) - DC_t A_T / A_t \right) \right)^+ \]

• With TUC: \( A_T \) is the only RV.
  • This is a simple European put option

• With PUC: use \( Z_T \) in place of \( Z_t \).
  • This is an exchange option.
$H(t) = \Psi_t + A_t$

- where $\Psi_t$ is in risk free bond, $A_t$ is in the DC assets (short).

$H_{bf}(t) = \Psi_{t-1}e^r + A_{t-1} \frac{A_t}{A_{t-1}}$

- is the value of the hedge brought forward.

Then the contribution at $t$ is

$$C_t = H(t) - H_{bf}(t)$$

- Adjust for survival.
So each month

- we re-value the hedge of the accrued benefit
- New hedge costs more (on average) because:
  - service is increased, salary is increased
  - random variation in old hedge value
- Contribution = cost of new hedge – value of old hedge
- Use Monte Carlo Simulation
How much does it cost?

Age 35 entrant, 1.5% accrual rate, 12.5% DC contributions
SO...

- The DB underpin appears affordable and attractive
- Surely should be a contender in the debate about pension design?
EXAMPLE 4

GUARANTEED MINIMUM INCOME BENEFITS

Claymore Marshall, Mary Hardy & David Saunders
GMXB

- GMXBs are embedded financial guarantees
- Suppose policyholder fund is $S_t$ at $t$.
  - GMMB guarantees fixed amount at maturity, $T$.
  - GMDB guarantees minimum sum on death before $T$.
- GMXBs are non-diversifiable, financial options,
- Use risk management tools from financial engineering.
THE GMIB

The GMIB rider offers max of

1. The accumulated fund minus fees: $S_f(T)$
2. Annuitize at a guaranteed rate, where the annuitized sum is the greater of:
   A. The initial single premium rolled up at a fixed rate (5% was common):
      \[ A(T) = S(0)(1+r_g)^T \]
   B. The greatest year end accumulated fund value over the contract term:
      \[ B(T) = \max \{ S_f(k): k=0,1,\ldots,T \} \]
THE GMIB

• That is, the payoff is the max of
  1. Accumulated fund less fees
  2. fixed 5% roll up, annuitized
     – guaranteed return component
  3. maximum year end accumulated fund, annuitized
     – maximum component
• And the fees depend on the potential amount annuitized.

“The most complex option ever contrived” (A White)
GMIB QUESTIONS

1. How much does this cost? Are the fees adequate?
   - Value using risk neutral simulation

2. What are the implications of this complex structure?
   - Analyse the contribution of the different components to the cost and risk


VALUING THE GMIB

\[
V(c) = E^Q \left[ \exp \left( - \int_0^T r(t) \ dt \right) \max\{BB(T)g a_{20}(T), S_f(T)\} \right]
\]

\[
dS(t) = r(t)S(t) \ dt + \sigma_S S(t) \ dW_S^Q(t)
\]

\[
d\hat{r}(t) = \alpha \{\Theta(t)/\alpha - r(t)\} \ dt + \sigma_r \ dW_r^Q(t),
\]

\[
BB(T) = \max(A(T), B(T))
\]

\[
g = \text{guaranteed annuity rate}
\]

\[
a_{20}(T) = \text{annuity value at maturity}
\]

\[
S_f(T) = \text{accumulated fund at maturity net of fees}
\]

Fee at \( t = c BB(t) \ dt \)
Fair fee rate, $1000 premium

<table>
<thead>
<tr>
<th>g</th>
<th>fair fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.50%</td>
<td>2%</td>
</tr>
<tr>
<td>6.50%</td>
<td>5%</td>
</tr>
<tr>
<td>&gt;7%</td>
<td>None</td>
</tr>
</tbody>
</table>
WHY DOES IT COST SO MUCH?

![Graph showing cost changes with fee rate and GMIB options.]

- GMIB
- GMIB without guaranteed return $r_g$
- GMIB with out running maximum

$g = 7.5\%$
Comments on pricing

- Fees charged appear low compared with risk neutral valuation
  - Reasons why we might have over-valued
    - lapses
    - irrational exercise
  - Reasons why we might have under-valued
    - longevity risk
    - fatter tailed distributions
    - Fund and timing options
Dynamic hedge is very complex

Examine a static hedge using realistic fee rates

- Static hedge assumes buy and hold
- Offer a range of instruments
- Minimize CTE (or Mean Square Hedging Error), with budget constraint.
OPTIMIZATION PROBLEM

• Assume transactions costs of 1% of each instrument

• Assume budget constraint
  • total investment = premium invested

• CTE optimization from Rockefeller and Uryasev (2000), Alexander, Coleman and Li (2006).
<table>
<thead>
<tr>
<th>GMIB</th>
<th>Mean HL</th>
<th>CTE 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>47</td>
<td>1072</td>
</tr>
<tr>
<td>Guaranteed Roll-up</td>
<td>89</td>
<td>302</td>
</tr>
<tr>
<td>P/h Fund</td>
<td>-184</td>
<td>0</td>
</tr>
</tbody>
</table>
ALL STOCK STATIC HEDGE

![Graph showing hedging loss at time T vs stock value at time T with various data points and labels: Maximum exercised, Guar. return exercised, Inv. account exercised, CTE contribution.](image)
MORE INSTRUMENTS

- Available Instruments:
  - European Puts, Calls with range of strikes
  - Zero Coupon Bonds with range of terms
- Optimized portfolio
  - Medium ZCB (-ve) Long ZCB (+ve)
  - Stock (>1)
  - Put options, strike $K=0.9S(0)$
OPTIONS, BONDS, STOCK HEDGE
ADD LOOKBACK OPTIONS

- Add lookback calls \((K=1.7S(0))\)
- Reduce Stock position \((0.15)\)
- No short bond position
OPTIONS, BONDS, LOOKBACKS, STOCK
HEDGING GMIBS – RESULTS I

<table>
<thead>
<tr>
<th>Hedging Error ($)</th>
<th>Mean</th>
<th>SD</th>
<th>VaR</th>
<th>CTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Only</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC1</td>
<td></td>
<td></td>
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<tr>
<td>PC2</td>
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</tr>
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<td>PC4B</td>
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</tr>
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<td>PS1</td>
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</tr>
<tr>
<td>PS2A</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>PS2B*</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

*Mean constrained to be 0
HEDGING GMIBS – RESULTS II

CTE Breakdown

- Hedging Error ($) vs. Hedging Component
- Stock Only vs. PC1, PC2, PC3, PC4A, PC4B*, PS1, PS2A, PS2B*

- HAV Component
- Guaranteed Return Component
- Investment Component

- Implications?
Some Conclusions

- For the premium we can either manage the lookback risk or the other risks; not both (static hedge results).
- The lookback risk is minor in pricing, but major in risk management.
- Removing either of the annuitization options offers a risk manageable product.
SUCCESS METRICS...

1. CTE Risk Measure:
   - Academic: B+
   - Technology Transfer: A+

2. RSLN Model:
   - Academic: A-
   - Technology Transfer: A+

3. Hybrid Pensions
   - Academic: B-
   - Technology Transfer: F*

4. GMIB
   - Academic: B
   - Technology Transfer: D
SUCCESS AND FAILURE IN KNOWLEDGE CREATION & MOBILIZATION

• Other professions integrate research and practice. What’s holding actuaries back?

• What can we do to improve?
  • as academics,
  • as practitioners,
  • as students
Academics

- Find **practical** solutions to **real** problems
- **Engage** with industry and the profession
- Willingness to **participate** in non-traditional communications
- Bring the **real world** **into the classroom**
- Use mathematics to bring clarity and precision
  - not for points scoring or snob value or obfuscation.
PRACTITIONERS, INDUSTRY, PROFESSION

- Develop an open mind to advances in actuarial science
- Support change agents -- to explain / bridge to advances
  - Place value on technical and business skills
- Fully engage in relevant CPD to improve skills and understanding.
- Support increased emphasis on rigorous, formal education of actuaries, through u-grad or post grad programs.

- **The Do-It-Yourself education model is a bad way to make doctors, lawyers, engineers or actuaries.**
STUDENTS

• Use your opportunities in formal education
  • you should be better educated than the self-trained exam passer.
  • what you are learning is real and relevant
• Don’t stop learning when you pass an exam or all exams
  • aim for deeper understanding of the foundational subjects.
  • Change will keep happening after you are a Fellow
• Be the change agents in your world.
Actuarial risk management is quantitative and qualitative, requiring a modern, complex, scientific tool kit, along with business strategic skills and professional integrity.

Academics and practitioners working together, developing rigorous and practical solutions to evolving challenges, will build better solutions and a safer financial sector for a more stable economy.

Nothing we do is more important than this.
QUESTIONS ... AND THANKS!

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