Complex Analysis Prelim Written Exam *Spring 2017*

*Questions are equally weighted. Give essential explanations and justifications: a large part of each question is demonstration that you understand the context and understand which issues are primary. Do not choose assumptions or contexts making the problems silly. Coherent writing is essential: your paper should not be a puzzle for the grader.*

Write your **codename**, not actual name, on each booklet. No notes, books, calculators, computers, cell phones, wireless, bluetooth, or other communication devices may be used during the exam.

[1] Write three terms of the Laurent expansion of \( f(z) = \frac{e^z - 1}{z(z-1)} \) centered at 0 and convergent in \(|z| > 1|\).

[2] Show that an \( \mathbb{R} \)-valued holomorphic function \( f \) is constant.

[3] Evaluate \( \int_{-\infty}^{\infty} \frac{e^{ix}}{1 + x^2} \).  

[4] Determine the radius of convergence of the power series for \( \frac{z^2}{1 - \cos z} \) at 0.

[5] Let \( f \) be an entire function such that \(|f(z)| \leq 1 + \sqrt{|z|}\) for all \( z \in \mathbb{C} \). Show \( f \) is constant.

[6] Show that there is a holomorphic function \( f \) on the region \(|z| > 2|\) such that \( f(z)^4 = z^4 + z + 1 \).

[7] Show that \( \frac{\pi^2}{\sin^2 \pi z} = \sum_{n \in \mathbb{Z}} \frac{1}{(z - n)^2} \)

[8] Make a change of coordinates to put the elliptic curve \( w^2 = z^4 + 1 \) into the (essentially) Weierstraß form \( y^2 = x^3 + bx + c \).  

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