Classify abelian groups of order 32.

For arbitrary prime $p$, construct a non-abelian group of order $p^3$.

Let $k$ be a field, $x$ an indeterminate. For distinct $a_1, \ldots, a_n$ in $k$, prove that there are unique $A_1, \ldots, A_n \in k$ such that
\[
\frac{1}{(x-a_1)(x-a_2)\ldots(x-a_n)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \ldots + \frac{A_n}{x-a_n} \quad \text{(in } k(x))
\]

Let $S, T$ be linear operators on a finite-dimensional complex vector space, with $ST = TS$. Show that $S$ and $T$ have a common eigenvector.

Show that the ideal generated by 13 and $x^3 - 2$ in $\mathbb{Z}[x]$ is maximal.

Let $R$ be a commutative ring with unit. Show that the collection of all nilpotent elements of $R$ is an ideal.

Show that $x^5 - 2$ is irreducible in $\mathbb{F}_{11}[x]$. 