

**N. Admal:** A Decomposition of the Atomistic Stress Tensor into an Elastic and a Plastic Component

In this talk, we propose an additive decomposition of the atomistic stress tensor into an elastic and a plastic part. Interestingly, the elastic part is independent of the choice of the potential energy representation. The continuum theory of elastoplasticity also provides an additive decomposition of the stress tensor into an elastic and a plastic part, which is based on the multiplicative decomposition of the deformation gradient. Through various numerical experiments we compare and highlight many similarities between the two decompositions.

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**P. Bauman:** Analysis of Defects in Nematic Liquid Crystals Described by Minimizers of De Gennes' Q-Tensor Energy

We investigate the structure of nematic liquid crystal thin films described by the Landau-de Gennes tensor-valued order parameter model with Dirichlet boundary conditions on the sides of nonzero degree. We prove that as the elasticity constant goes to zero in the energy, a limiting uniaxial nematic texture forms with a finite number of defects, all of degree  $1/2$  or  $-1/2$ , corresponding to vertical disclination lines at those locations.

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**P. Belik:** Fractal Powers in Serrin's Swirling Vortex Solutions

We consider a modification of the fluid flow model for a tornado-like swirling vortex developed by J. Serrin in 1972 where velocity decreases as the reciprocal of the distance from the vortex axis. Recent studies, based on radar data of selected severe weather events, indicate that the angular momentum in a tornado may not be constant with the radius, and thus suggest a different scaling of the velocity/radial distance dependence.

Motivated by this suggestion, we consider Serrin's approach with the assumption that the velocity decreases as the reciprocal of the distance from the vortex axis to the power  $b$  with a general  $b > 0$ . This leads to a boundary-value problem for a system of nonlinear differential equations. We analyze this problem for particular cases, both with nonzero and zero viscosity, discuss the question of existence of solutions, and use numerical techniques to describe those solutions that we cannot obtain analytically.

This is joint work with D. Dokken, K. Scholz and M. Shvartsman.

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**L. Caffarelli:** Homogenization of a Front Driven by Forcing and Curvature

In joint work with Regis Monneau, we study a front propagating due to a combination of its curvature and a periodic forcing. In some cases we show existence of effective speeds, in others, lack of homogenization.

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### C. Calderer: James Serrin, The Applied Mathematician

In a tribute article\*, *Applied Mathematics and Scientific Thought*, James Serrin states that *the mathematical work of Lamberto Cesari has always had a strong element of concrete reality running through it*. In this article, Serrin sets out his vision and thoughts on *the nature of mathematics as it is related to the physical world*. This is also Serrin's definition of *applied mathematics*. It is within this context, that he gives praise as well as cautionary advice on the use of the term *modeling*: *... it carries a certain pedestrian tone, subtly lowering the intellectual level*.

This article appeared in 1982, also the inaugural year of the *Institute for Mathematics and its Applications*. Under its founding director, Hans Weinberger, the IMA soon became the world hub of applied mathematics, where Serrin's vision and organizational activities met with those of many other renowned scientists and mathematicians. In that framework, the concept of *modeling* grew into many new and exciting directions, and spanned many fields. In the field of mechanics, Jerry Ericksen led a vigorous research initiative that met with significant progress in nonlinear analysis, especially in the field of liquid crystals.

Works by James Serrin, especially some published in the 1980's and 1990's, remain highly relevant to address some of the challenges posed by current liquid crystal research. In this presentation, I will explore mathematical connections between some topics of Serrin's research, *Variational Problems for Minimal Surfaces, Fluid Interfaces and Phase Transitions, Liquid Mixtures and the Prandl Boundary Layer Theory* and current problems of liquid crystals. These include *defects, colloidal and chromonic systems, and elasticity*. The latter arise, for instance, in applications to the modeling and design of nanofluidic devices and clustering phenomenon in biology.

\* *Nonlinear Analysis and Optimization*, Ed, C. Vinti, Lecture Notes in Mathematics, no. 1107, (1982), pages 19-27, Springer-Verlag.

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### I. Fonseca: Variational Methods in Materials and Image Processing

Several questions in applied analysis motivated by issues in computer vision, physics, materials sciences and other areas of engineering may be treated variationally leading to higher order problems and to models involving lower dimension density measures. Their study often requires state-of-the-art techniques, new ideas, and the introduction of innovative tools in partial differential equations, geometric measure theory, and the calculus of variations.

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**E. Fried:** Stability and Bifurcation of Flexible Filaments Spanned by Fluid Films

A class of variational problems describing the equilibrium of a flexible filament spanned by a fluid film will be presented and discussed, with an emphasis on understanding the stability and bifurcation of flat, circular configurations. Chief among these problems is that involving the Kirchhoff-Canham-Helfrich functional, which arises on modeling as an inextensible filament endowed with elastic resistance to bending and twisting and endowing the fluid film with surface tension and elastic resistance to bending. Various approaches to the singular limit that arises on neglecting the bending resistance of the fluid film, for which the spanning surface must have zero mean curvature, will also be considered.

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**M.E. Jabbour:** Surface Instabilities during Step-flow Epitaxy. Adatom Electromigration and Elastic Step Interactions Revisited

A thermodynamically consistent theory for step-flow epitaxy that incorporates adatom electromigration and elastic step interactions is presented. The stability analysis of the resulting moving-boundary problem sheds new light on the onset of surface instabilities that remain unaccounted for by the standard Burton-Cabrera-Frank model.

This is joint work with N. Kirby and E. Fried.

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**C. Kenig:** Soliton Resolution for Nonlinear Wave Equations

We discuss the recent proofs of soliton resolution for radial solutions of the energy critical wave equation in 3 spatial dimensions (with Duyckaerts and Merle), a case in which the decomposition is unstable, and for exterior co-rotational wave maps from  $R^3$  with the unit ball removed, and Dirichlet conditions, into  $S^3$  (with Lawrie and Schlag) a case in which the decomposition is stable. Both proofs use crucially the “channel of energy property” introduced by Duyckaerts, Kenig and Merle.

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**N. Kirby:** The Gamma-limit of a Model for the Elastic Energy of an Inextensible Ribbon

In 1930, Sadowsky proposed the limiting form of the energy of an inextensible ribbon of negligible width. Sadowsky and subsequent authors based this limit upon taking a pointwise limit of the integrand for a ribbon with positive width, where the energy is written as an integral along the centerline of the ribbon. Subsequent to these contributions the method of gamma-convergence has matured significantly. It will be demonstrated that the Sadowsky's proposed functional is the proper gamma-limit of the energy functionals for inextensible ribbons with fixed centerline upon passing to the limit of a ribbon of negligible width.

This is joint work with E. Fried.

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**S. Mayboroda:** Localization of Eigenfunctions

The property of the localization of the eigenfunctions in rough domains or rough materials permeates acoustics, quantum physics, elasticity, to name just a few. Localization on fractal domains was used for noise abatement walls which up to date hold world efficiency record. Anderson localization of quantum states of electrons has become one of the prominent subjects in quantum physics, harmonic analysis, and probability alike. Yet, no methods could predict specific spatial location of the localized waves.

In this talk I will present recent results which demonstrate a universal mechanism governing localization of the eigenfunctions of an elliptic operator. We prove that for any operator on any domain one can reveal a “landscape” which splits the domain into disjoint subregions. Starting from this landscape, we recover location, shapes, and frequencies of the localized eigenfunctions of low energy, and describe the effects of delocalization taking place as energy increases.

This is joint work with D. Arnold, G. David, M. Filoche, and D. Jerison.

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**R. Paroni:** Plate Theory for Inhomogeneous, Anisotropic, Linearly Elastic Bodies as a Variational Limit of the Complementary Energy Functional

In the last three decades the principle of the minimum potential energy has been used, in conjunction with the theory of Gamma-convergence, to justify/derive models for thin structures starting from the three-dimensional theory. We here consider a inhomogeneous, anisotropic, linearly elastic body whose reference configuration is a cylinder of height  $h$ . Then, by means of Gamma-convergence, we study the asymptotic behavior, as  $h$  goes to zero, of the sequence of complementary energies. This approach has the advantage to deliver a direct approximation of the stress field.

This is a joint ongoing work with Francois Murat (Univ. Paris VI).

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**L.A. Peletier:** Rational Pharmacology

Whereas the basic models of mechanics can boast a centuries old history and the best of pedigree, the same cannot be said of pharmacology, the branch of medicine and biology concerned with the study of drug action. The underlying phenomena are complex and the road from administration of drug to resulting effect, the *biomarker*, is long and contorted. In this lecture we discuss two case studies which display the complexities of the subject, but also the opportunities for the type of reasoning advanced in rational mechanics.

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**P. Podio-Guidugli:** Some Thoughts about the Format of Continuum Thermodynamics

To Be Announced

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**P. Pucci:** Existence of Entire Solutions for Elliptic Problems Involving the Fractional Laplacian

We study the existence and multiplicity of entire solutions for elliptic equations, driven by a non-local integro-differential operator, which main prototype is the fractional Laplacian. The model under consideration, denoted by  $P_\lambda$ , depends on a real parameter  $\lambda$  and involves two superlinear nonlinearities, one of which could be critical or even supercritical. The main theorem of the paper establishes the existence of three critical values of  $\lambda$  which divide the real line in different intervals, where  $P_\lambda$  admits no solutions, at least one nontrivial non-negative entire solution and two nontrivial non-negative entire solutions (in Journal of Differential Equations, **255** (2013), 2340-2362, joint work with G. Autuori).

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**B. Seguin:** Stability of a Soap Film in a Cylindrical Tube

Recently, Cox and Jones studied an interesting variant of the classical Plateau problem involving a soap film confined to a cylindrical tube. Through experiments and some analysis, they found that the dimensions of the tube strongly influence the equilibrium shape of the confined soap film. In this talk, an area minimization problem associated with determining the shape of the film will be formulated and analyzed to determine which surfaces are local minima. The connection between a functional inequality and the associated eigenvalue problem plays an important role in the analysis. One finding that is of interest is that more candidates for local minima exist when the length of the cylinder is small compared to its radius than when the length is large compared to its radius.

This is joint work with E. Fried.

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**M. Šilhavý:** Phase Transitions with Interfacial Energy: Interface Null Lagrangians, Polyconvexity, and Existence

The paper proves the existence of equilibrium two phase states with elastic solid bulk phases and deformation dependent interfacial energy. The states are pairs  $(y, E)$  consisting of the deformation  $y$  on the body and the region  $E$  occupied by one of the phases in the reference configuration. The bulk energies of the two phases are polyconvex functions representing two wells of the substance. The interfacial energy is interface polyconvex. The last notion is introduced and discussed below, together with the interface quasiconvexity and interface null Lagrangians. The constitutive theory and equilibrium theory of the interface are discussed in

detail under appropriate smoothness hypotheses. Various forms of the interfacial stress relations for the standard and configurational (Eshelby) interfacial stresses are established. The equilibrium equations are derived by a variational argument emphasizing the roles of outer and inner variations.

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#### **V. Sverak:** On Serrin's Weak-Strong Uniqueness Theorem for the Navier-Stokes Equation

In 1930s Leray introduced the notion of weak solutions for the Navier-Stokes equations and showed that the classical Cauchy problem has at least one global solution in this class. However, the uniqueness of the solutions has not been established. Around 1960, extending earlier works of several authors on the subject, Serrin identified a class of solutions for which uniqueness can be proved. We will discuss recent developments concerning these topics, which suggest that Serrin's assumptions are close to being optimal.

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#### **E. Virga:** Curvature Potentials on Nematic Shells

Nematic shells are thin films of nematic liquid crystal deposited on rigid colloidal particles, which can be manufactured in different shapes and guises. The two-dimensional order tensor that describes the local organization of liquid crystal molecules, which tend to lie parallel to the colloids' surface, vanishes wherever no orientation is prevailing on average. The points where this takes place are called defects, as they lack order. The lecture will review recent work concerned with the interaction between defects and the underlying surface. In particular, arguments will be offered that identify appropriate geometric potentials, depending on the shell's shape, which either promote or hamper defects, attract or repel them.

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