Load software & enter the PDE of interest \( u_t = v_x, v_t = B(u) \ u_x + C(u) \)

```maple
> restart; with(DEtools): with(PDEtools):
> telpot := [diff(u(x,t),t) = diff(v(x,t),x), diff(v(x,t),t) =
B(u(x,t))*diff(u(x,t),x) + C(u(x,t))];

\[
telpot := \left[ \frac{\partial}{\partial t} u(x,t) = \frac{\partial}{\partial x} v(x,t), \frac{\partial}{\partial t} v(x,t) = B(u(x,t)) \left( \frac{\partial}{\partial x} u(x,t) \right) + C(u(x,t)) \right]
\] (1.1.1)

> declare(telpot, quiet):
> telpot;

\[
\left[ u_t = v_x, v_t = B(u) \ u_x + C(u) \right]
\] (1.1.2)
```

Generate the symmetry defining system of \( u_t = v_x, v_t = B(u) \ u_x + C(u) \)

Automatically generate the symmetry defining system for symmetry vector fields

```maple
> telsys := DeterminingPDE(telpot, [u, v], [xi(x, t, u, v), tau(x, t, u, v), eta(x, t, u, v), zeta(x, t, u, v)], integrabilityconditions = false):

> declare(telsys, quiet):
> telsys;

\[
\begin{align*}
\{-\tau_u + \xi_x \tau_u - \xi_u + \tau_v B, & -\tau_v B^2 + B \xi_x, \eta_t - \xi_x + \tau_x C + \eta_v C, \xi_x - \xi_u + \eta_u - \xi_v - \xi_v, \\
& + \tau_x B - \xi_v C + \eta_v B + \tau_v C - \xi_u - \tau_v B + B \xi_x - \eta B_u - B \eta_v - 2 \tau_v B C + \xi_v B, \\
& -\tau_v C + \xi_u - \xi_u - \eta_v B + \tau_x B - \xi_v C, \xi_v C - \tau_t C - \tau_v C^2 - B \eta_x - C u + \xi_q \}
\end{align*}
\] (1.2.1)
```

Simplify the defining system using rifsimp

Note that you can either do a complete analysis directly using rifsimp, or just ask for components greater than a specified dimension using the command maxdimsystems. This is very useful in challenging research problems, where computing all components maybe very challenging. Note we have also excluded the cases where B'(u) and C'(u) are zero.

```maple
> telsysineq := [op(telsys), diff(B(u), u) \neq 0, diff(C(u), u) \neq 0]:
```
Notice we only looked for infinite dimensional groups above. When the computation encountered a split in the classification tree - for example - a condition on C, B where the dimension of the group became finite - it was discontinued. Such incomplete cases are represented by the green branches in the tree above.

The initial data for the infinite case 6 above is given below.

> `initialdata(rsys[6][Solved]);`
More detailed look at interesting cases

\[
\begin{align*}
\text{table} \left( & \text{Infinite} = \left[ \eta(x_0, t_0, u_0, v) = F_1(v), \tau(x_0, t_0, u_0, v) = F_2(v) \right], \text{Finite} = \left[ B(u_0) = C1, C(u_0) = C2, D(C)(u_0) = C3, D_2(\tau)(x_0, t_0, u_0, v_0) = C4, \xi(x_0, t_0, u_0, v_0) = C5, D_4(\xi)(x_0, t_0, u_0, v_0) = C6, \zeta(x_0, t_0, u_0, v_0) = C7 \right] \right) \\
& \text{print}(\text{rsys}[6]); \\
& \text{table} \left( \text{dimension} = \infty, \text{Solved} = \left[ \tau_{t, t} = 0, \tau_{t, v} = -\frac{1}{2} \frac{C_u (\xi_v C + \eta_v B)}{CB}, \xi_{v, v} = -\frac{\eta_{v, v} B}{C} \right], \left[ \xi_x = \frac{\tau_x C + \eta_x C_u + \tau_x C^2}{C}, \tau_x = \frac{\xi_x C - \eta_x B}{B}, \eta_x = \frac{-\eta C_u - \tau_x C - \tau_x C^2}{B}, \xi_x = \frac{(\xi_v C + \eta_v B) C}{B}, \xi_{v, v} = -\frac{1}{2} \frac{\eta_v B}{\xi_v C}, \eta_x = \frac{1}{2} \frac{\left( \xi_v C + \eta_v B \right) C}{B}, \xi_x = \xi_x, \eta_x = -\frac{\eta C_u - \tau_x C^2}{C}, \xi_x = \xi_x C + \eta_x B, \xi_{v, v} = 0, C_{u, u} = \frac{2 C_u}{C}, B_u = \frac{2 B C_u}{C} \right], \text{Pivots} = \left[ B \neq 0, C_u \neq 0 \right], \text{Case} = \left[ \left[ 2 B C_u - C B_u = 0, \tau_x \right], \left[ C C_{u, u} - 2 C_u = 0, \eta_x \right] \right] \right)
\end{align*}
\]

> \>

> \>

> dsol := \text{dsolve} \left( \left[ \text{rsys}[6][\text{Solved}][[-1], \text{rsys}[6][\text{Solved}][[-2]], \left[ B(u), C(u) \right] \right); \\
dsol := \left[ \left\{ C = -\frac{1}{C_2 u + C_3} \right\}, \left\{ B = C1 C^2 \right\} \right] \right)

> >

> >

> >

> Now consider symmetry groups of dimension \( \geq 4 \)

> Initially just put mindim = infinity

> rsys := \text{maxdimsystems}(\text{telsysineq}, \text{vars}, \text{mindim} = 4, \text{output} = \text{rif});

> caseplot(rsys, \text{vars});

Case 3: 4-d
Case 5: 4-d
Case 7: 4-d
Now we get a much more detailed classification tree. Here we see a highly nontrivial case, discussed in Reid 91.

> print(rsys[14]);

\[
\text{table}: \begin{cases} \text{dimension} = 4, \text{Solved} = & \xi_x = -\frac{\eta \left( C C_{u,u} - 2 C_u^2 \right)}{C C_u}, \tau_x = 0, \eta_x = 0, \zeta_x = 0, \xi_x = 0, \tau_x = 0, \eta_x = 0 \end{cases} \tag{1.5.1}
\]

\[
\begin{align*}
= -\frac{\eta \left( 2 C_u^4 - C_{u,u}^2 C^2 - 2 C_u^2 C C_{u,u} + C^2 C_{u,u,u,u} C_u \right)}{C C_u \left( C C_{u,u} - 2 C_u^2 \right)}, & \eta_x = 0, \zeta_x = 0, \xi_x = 0 \\
= \frac{\eta \left( -2 C_{u,u}^2 C + C_{u,u} C^2 + C C_{u,u,u,u} C_u \right) B}{C C_u \left( C C_{u,u} - 2 C_u^2 \right)}, & \tau_u = 0, \eta_u = 0
\end{align*}
\]
\[ -\eta \left( \frac{2C_u^4 - C_{u,u}^2 C^2 - 2C_u^2 C_{u,u} + C^2 C_{u,u,u,u} C_u}{C C_u (C C_{u,u} - 2 C_u^2)} \right), \eta_v = 0, \xi_v = 0, \tau_v \]

\[ = \frac{(-2 C_{u,u}^2 C + C_{u,u} C_u + C^3 C_{u,u,u,u} C_u)}{C C_u (C C_{u,u} - 2 C_u^2)} \eta, \eta_v = 0, \xi_v = -\frac{\eta (C C_{u,u} - 2 C_u^2)}{C C_u}, \]

\[ C_{u,u,u,u} = \frac{1}{-C_u C_u^2 C_{u,u} + 2 C_u^3 C} \left( 12 C_{u,u,u} C_{u,u,u} C_{u,u} - 4 C_{u,u,u} C_u - 12 C_u C_{u,u,u} C_{u,u,u} C_u \right), B_u = \frac{2 B C_u}{C}, \text{Pivots} \]

\[ = \left[ B \neq 0, C C_{u,u} - 2 C_u^2 \neq 0 \right], \text{Case } = \left[ [2 B C_u - C B_u = 0, \tau_v], [C C_{u,u} - 2 C_u^2 \neq 0, \eta_v] \right] \left[ -12 C_{u,u,u} C_{u,u} C_{u,u,u} C_{u,u} + 4 C_{u,u,u} C_u^3 + 12 C_{u,u,u} C_u^3 C - 6 C_{u,u,u} C_u^3 \right. \]

\[ - C_{u,u,u}^2 C_{u,u}^2 - C_{u,u,u}^2 C_{u,u,u} C_{u,u,u} + 2 C_{u,u,u} C_{u,u,u} C_{u,u,u} + 2 C_{u,u,u} C_{u,u,u} C_{u,u,u} = 0, \eta ] \]

\[ \]

\[ > > \]

\[ > > rsys := \text{maxdimsystems}(\text{telsysineq, vars, mindim = 0, output = rif}) : \]

\[ > > \text{caseplot}(rsys, \text{vars}); \]

\[ \text{Case 1: 3-d} \]
\[ \text{Case 2: 3-d} \]
\[ \text{Case 3: 4-d} \]
\[ \text{Case 4: 3-d} \]
\[ \text{Case 5: 4-d} \]
\[ \text{Case 6: 3-d} \]
\[ \text{Case 7: 4-d} \]
\[ \text{Case 8: 3-d} \]
\[ \text{Case 9: 4-d} \]
\[ \text{Case 10: 3-d} \]
\[ \text{Case 11: 4-d} \]
\[ \text{Case 12: infinite} \]
\[ \text{Case 13: 3-d} \]
\[ \text{Case 14: 4-d} \]
\[ \text{Case 15: infinite} \]
Rif Case Tree

```
(1.5.1) (1.5.1) (1.3.1) (1.3.1)
  3-d 1
  3-d 2
  4-d 3
  3-d 4
  4-d 5
  3-d 6
  4-d 7
  3-d 8
  4-d 9
  3-d 10
  4-d 11
  (inf.) 12
  3-d 13
  4-d 14
  (inf.) 15
```

```
= = = = = =
Rif Case Tree
```