**SOME PROBLEMS AND PROGRAMS (DISORDERED)**

(1) (Greenlees-May) Generalize the localization and completion theorem for $MU_G$-homology and cohomology to the case of arbitrary compact Lie groups. We proved it for finite extensions of tori, but it should hold in general. This is the kind of situation for which transfer arguments were invented, but we could never make it go.

(2) (Greenlees-May) Conjecture: The Segal conjecture holds for compact Lie groups, provided that one understands completion with respect to the augmentation ideal of the entire ring or Mackey functor $\pi_G^*$ and not just the Burnside ring. This is not strictly true as stated, but something along these lines should hold.

(3) Probably the second problem and possibly the first will require an honest treatment of completions at infinitely generated ideals.

(4) Study $MU_*^G$: Global Mackey functors; equivariant formal group laws; detailed study of Abelian groups, starting with circle and cyclic groups. The conjecture that $MU_*(BG)$ is free over $MU_*$ on generators of even degree is rendered implausible but not disproven by Kriz's counterexamples for Morava $K$-theories. It would be interesting to see what happens in those examples, through explicit calculations.

(5) (French) Understand equivariantly more of Adams' fundamental work on the image of $J$. The reformulation of Adams' work using information on classifying spaces and the Adams conjecture gives an outline of what the global picture should look like. The best available version of the Adams conjecture, which must be fundamental, is due to McClure, but it is not well understood on the classifying space level. In general, understand the basic chromatic level one phenomena equivariantly.

(6) (Westerland did a lot). Understand chromatic theory geometrically on the space and not just the spectrum level, analogous to the space level understanding at chromatic level one.

(7) Sample calculations of the proper subgroup family case of Amitsur-Dress-Tate cohomology should be of interest. Related to the Atiyah-Hirzebruch-Tate spectral sequence for stable cohomotopy. In particular, this should lead to sample calculations of root invariants in the generalized alternative form given by Greenlees and May (Tate).

(8) Various questions concerning the structure of the Tate spectrum associated to connective equivariant $K$-theory; related conjectures of periodicity lowering involving the higher $BP\langle n\rangle$ spectra. (Ando, Morava, Sadofsky, Greenlees and others are better informed about this).

(9) (McClure, Kriz, Basterra, Mandell) Better understanding of $BP$ and related spectra.

(10) (Kriz–May) Conjecture: The derived category of a DGA admits an embedding as a full subcategory of an Abelian category.

(11) (Mandell) Many questions about $p$-adic homotopy theory. Develop a theory of formality. Define Hopf $E_\infty$ algebras, relate them to $H$-spaces, and in particular see if this helps to understand $p$-compact Lie groups in the sense of Dwyer and Wilkerson, etc.

(12) (Hesselholt, Madsen, May; likely obsolete) Use orthogonal $G$-spectra to obtain a clearer conceptual understanding of topological Hochschild and cyclic homology. There should be an equivariant version of TC, and it might be relevant to Galois extensions. Give a model category of cyclotomic orthogonal $S^1$-spectra.

(13) Detailed study of the multiplicative structure and uniqueness of module spectra over $MU$ with standard coefficient rings. There is a mess of a literature, and the modern substitute for Baas–Sullivan theory should allow clarification.

(14) Similarly, study $MSp$-module spectra using modern technology. There is a large and messy literature using Baas–Sullivan theory.

(15) Multiplicative equivariant infinite loop space theory (in progress).

(16) Fully develop equivariant algebraic $K$-theory (in progress)

(17) Develop the Adams spectral sequence for modules over $R$-algebras, where $R$ is an $E_\infty$ ring spectrum and algebras and their modules are understood homotopically, with structure maps in the derived category of $R$-module spectra. Flatness is the problem.

(18) Similarly revisit equivariant ASS (old paper by Greenlees).

(19) (Costenoble, May, Waner) Equivariant orientation theory is strange and complicated. Still, it has been developed, and there are many questions about the relationships among different equivariant versions of the Thom isomorphism and Poincaré duality theorems. An essential point is to relate the geometric version (CMW) with cohomological versions (such as May's).

(20) (Lewis) Take Lewis's work on the algebra of Mackey ring functors (Green functors) seriously. One can define Spec, and one can define schemes. One should be able to develop an algebraic geometry of schemes over Mackey functors, or over more general rings with many objects.

(21) Compute almost anything equivariant! It was a stroke of luck that only very simple calculations were needed for Kervaire.

(22) For example, calculate equivariant characteristic classes even just rationally for cyclic groups $G$, complex $G$-bundles and, say, representation ring coefficients.

(23) For example, calculate the $RO(G)$-graded cohomology of a point for some non-abelian groups $G$, say the dihedral group of order 6 for a start. But any small finite groups would be interesting.

(24) For example, calculate (even with constant coefficients in $\mathbb{F}_2$ the Bredon homology of $\Omega^V\Sigma^V X$ for obvious choices of $G$-representations $V$. Is there a sensible notion of $G$-braid groups?