1. **Structured ring spectra p-divisible groups.** A map $\mathcal{M} \to \mathcal{M}_{\mathbf{fg}}$ from an algebraic stack to the moduli stack of formal groups is realizable if there is a sheaf $\mathcal{O}$ of E_∞-ring spectra on $\mathcal{M}$ (in an appropriate topology) with $\pi_*\mathcal{O} \cong \omega^{\otimes */2}$, where ω is the sheaf of invariant differentials.

(a) It has been asserted that it's not possible to realize $\mathcal{M}_{\mathbf{fg}}$ itself, mostly because the only sensible topology is the *fpqc*-topology. It would nice to have this fact proved and recorded, if it's true.

(b) Results of Lurie give realization criteria for étale maps $\mathcal{M} \to \mathcal{M}_p(n)$ where $\mathcal{M}_p(n)$ is the moduli stack (over $\mathbb{Z}_p$) for p-divisible groups of height n with formal part of dimension 1. Explore the geometry of the map $\mathcal{M}_p(n) \to \mathcal{M}_{\mathbf{fg}}$ to the moduli stack of formal groups. Understand the resulting descent problem as well. The map is not representable if $n > 1$.

(c) All E_∞-ring spectra come equipped with power operations. Lurie's result and its antecedent, the Hopkins-Miller theorem for Morava E-theories, produce E_∞-ring spectra but make no mention of power operations; thus it would seem the power operations are canonically dictated by the geometry of p-divisible groups. Explain this. Recent work of Rezk would be a place to start; this suggests analysis of the subgroup structure of p-divisible groups is important.

(d) Shimura varieties. One of the important features of the Hopkins-Miller theory of topological modular forms is that one gets sheaf of E_∞-ring spectra on the *compactified* Deligne-Mumford moduli stack of elliptic curves. Are there good compactifications of Shimura varieties that would apply to the Behrens-Lawson theory of topological automorphic forms? Work of Kai-Wen Lan might be applicable.

2. **Calculations in $K(2)$-local homotopy theory.** Much has been done, but more can be understood. For example, uses Olivier's thesis (Strasbourg 2013) to understand the Shimomura school's calculations at large primes.

3. **Chromatic Assembly.** The chromatic fracture square reads

$$\begin{tikzcd} L_n X \arrow[r] \arrow[d] & L_{K(n)} X \arrow[d] \\ L_{n-1} X \arrow[r, "j"'] & L_{n-1} L_{K(n)} X. \end{tikzcd} \tag{1}$$

(a) Assuming we know $L_{K(n)}X$, give an effective procedure—or even a systematic set of examples—for computing $L_{n-1}L_{K(n)}X$; equivalently, given some way to understand the nth monochromatic layer M_nY where Y is $K(n)$-local. As a point of entry, first study $L_{K(n-1)}L_{K(n)}X$.

Some preliminary algebraic work has been done in the papers of Torii (MR2004428), which uses work of Gross to study what happens to deformations of formal groups. For example, given a height n formal group G over an algebraic extension $\mathbb{F}$ of $\mathbb{F}_p$, we can take the deformation of G to $\mathbb{F}[[u_{n-1}]]$ and then consider this deformation as a height $n-1$ formal group over $\mathbb{F}[[u_{n-1}]][u_{n-1}^{-1}]]$. We might examine and reinterpret this algebra in homotopy theory and then extend this to a calculation and interpretation of

$$(E_{n-1})_*E_n = \pi_*L_{K(n-1)}(E_{n-1} \wedge E_n)$$

where E_s is Morava E-theory.

This is surely naive. Write C_n for the cyclic group. Then the formal group over $(E_n)_*$ is the formal spectrum of

$$\pi_0F(\mathbb{C}\mathrm{P}^\infty, E_n) = \lim \pi_0F(BC_{p^n}, E_n)$$

and we probably should consider the inverse system

$$\pi_0L_{K(n-1)}F(BC_{p^n}, E_n)$$

which is a p-divisible group. Thus p-divisible groups appear again.

(b) The Chromatic Splitting Conjecture asserts that the map j of (1) is a split inclusion—and much more besides. Can even this simple statement be verified, at least for some X?

(c) Revisit the Chromatic Splitting Conjecture. If it's true, an initial case to study would be when n is very small with respect to the prime p. Are there cases where it might not true? For example, if n is not a unit in p, the reduced norm map $\mathbb{G}_n \to \mathbb{Z}_p$ from the Morava stabilizer group doesn't have a central splitting. if $\mathbb{G}_n^1$ is the kernel of the reduced norm, there is then a Lyndon-Serre-Hochschild Spectral Sequence

$$H^s(\mathbb{Z}_p, H^t(\mathbb{G}_n^1, E_*X)) \Longrightarrow H^{s+t}(\mathbb{G}_n, E_*X))$$

and the action of $\mathbb{Z}_p$ can be very complicated. Does that twist the cohomology so that the CSS becomes difficult? Recent work of Beaudry might be applicable.

4. **Gross-Hopkins duality.** Brown-Comenetz duality and its variants are something of a curiosity in stable homotopy theory, but it is an insight of Hopkins that in the $K(n)$-local category it is much more like Serre-Grothendieck duality.

(a) Flesh out that statement. Some work has been done by Hopkins himself, and more by Devinatz, but this is just the start.

(b) If possible, compare Gross-Hopkins duality to the Poincaré duality structure of the Morava stabilizer group. Again some work has been done by people looking at Shimomura's calculation, most lately by Behrens. Develop a larger theory that works, if possible, at primes where there's torsion in the Morava stabilizer group.

(c) Think about the Behrens $Q(\ell)$ spectra in this context.

(d) I don't know this for a fact, but Lurie's theory probably gives a notion of Serre-Grothendieck duality in derived algebraic geometry. Make this precise and concrete; that is, give calculations. Work of Stojanoska should point the way.

5. **Go Equivariant.** Equivariant formal group laws and equivariant complex cobordism have been around for a while, but it may be time to take it up a notch. Recent work of Abrams-Kriz give a place to start.

(a) Think about the Abrams-Kriz work; they give a calculation, but it might be fruitful to think about that calculation in terms of the functors represented by these calculated rings—a sort of interpretation Lazard ring interpretation. There is some unpublished work of Greenlees (on his home page) worth reading.

(b) Abrams-Kriz works only for finite abelian groups, mostly because they have good duals. Is there a good notion of global equivariant formal group laws, in the fashion on Bohmann and Schwede? Does it good interpretation algebraically (Is there a Lazard object?) or homotopically?