Math 5285H

Final Exam

No collaboration is allowed. This test is open-book and open-library but no electronic sources may be consulted.

This test is due in-class on **Wednesday, December 16**.

1. Classify all groups of order 51.
2. Classify all groups of order 39.
3. Describe one of the Sylow p-subgroups of $GL_2(\mathbb{Z}/p)$.
4. Let G be a group and $Aut(G)$ be the automorphism group of G, i.e.

   $$Aut(G) = \{\text{bijective group homomorphisms } \phi : G \to G\}$$

   where the multiplication is given by $(\phi, \psi) \mapsto \phi \circ \psi$.

   For each $x \in G$, there is an *inner automorphism* $c_x \in Aut(G)$ given by $c_x(g) = xgx^{-1}$. Show that the set

   $$Inn(G) = \{c_x | x \in G\}$$

   is a normal subgroup of $Aut(G)$.
5. Suppose p, q, and r are primes with $p > q > r$. Show that any group of order pqr has a normal Sylow subgroup. (Hint: Count things.) Bonus points for showing that the Sylow p-subgroup itself is normal.