

The main themes covered in the course will be motivated by the study of Partial Differential Equations of Fluid Mechanics. The motion of fluids is described by the Navier-Stokes equations or Euler's equations, and the study of some aspect of these equations belongs to the Theory of Partial Differential Equations. However, the behavior of Navier-Stokes solutions is complicated, and to understand them better, it seems to be necessary to bring in methods from areas such as Statistical Mechanics, Geometrical Mechanics and Symplectic Geometry, Lie Group theory, etc. In this class we will study the motion of fluids from all those perspectives. No previous knowledge of these topics will be assumed, but the participants should have some knowledge of Ordinary and Partial Differential Equations, at the level of an introductory graduate course. Knowledge of elementary Functional Analysis would be useful, but it is not necessary - the few results we will possibly need can be learned along the way. Open problems of various level of difficulty will be mentioned. The grade will be based on homework assignments.

Course notes will be provided. I will follow no particular textbook, but relevant literature will be discussed.

Topics will include:

The basics of Fluid Mechanics,

The Geometry of Euler's Equations (Hamiltonian Mechanics for system with symmetries, Symplectic reduction, Finite-dimensional groups with left-invariant metric, diffeomorphism groups and Euler's equations),

Statistical Mechanics of 2d Euler's Equation, (Basic Ideas of Statistical Mechanics, their application to 2d fluids, long-time behavior of solutions of Euler's equations.)

Kolmogorov's Theory of turbulence.

What does all this say about the Navier-Stokes and Euler solutions? (Illustration of behavior of Navier-Stokes and Euler solutions, known results in PDE analysis of the Navier-Stokes and Euler's equations.)

Model equations. (Simple model equations which are more manageable than Euler/Navier Stokes can bring important insights into these problems and will be discussed at various points.

Currently the class is scheduled for the MWF 1:25-2:15 slot.