

Math 3593H Honors Math II
Quiz 3, Thursday March 23, 2017

Instructions:

20 minutes, closed book, no electronic devices,
but an 8.5×11 page of notes is OK.

There are three problems, worth a total of 20 points.

1. (9 points)

Let $A \subset \mathbb{R}^2$ be the region bounded

- above by the parabola $y = x^2$,
- below by the x -axis,
- on the right by the vertical line $x = 1$.

Compute

$$\int_A xy |dxdy| \left(= \int_{\mathbb{R}^2} xy \cdot 1_A(x, y) |dxdy| \right).$$

(Hint: it's always a good idea to sketch A first.)

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2. (6 points)

What is the volume of the image of the unit cube $Q = [0, 1]^3 \subset \mathbb{R}^3$ under the linear transformation $\mathbb{R}^3 \xrightarrow{T} \mathbb{R}^3$ defined by

$$T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}, \quad T(\mathbf{e}_2) = \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}, \quad T(\mathbf{e}_3) = \begin{bmatrix} 16 \\ -73 \\ 3 \end{bmatrix}?$$

3. (5 points)

Prove or disprove: the subset $\mathbb{Q}^2 \subset \mathbb{R}^2$ consisting of all points with rational coordinates has measure zero.