Math 3593H Honors Math II
Midterm exam 2, Thursday April 6, 2017

Instructions:
50 minutes, closed book, no electronic devices, but an 8.5×11 page of notes is fine. There are four problems, worth 25 points each.

1. (25 points) Find the coordinates $(\bar{x}, \bar{y})$ for the centroid (=center of gravity) of the subset $A \subset \mathbb{R}^2$ bounded above by the curve $y = x^3$, bounded below by the x-axis, bounded on the right by the line $x = 1$.

Half credit for setting up the two integrals, half for evaluating them. (Hint: sketch A first!)

2. For these two problems, set up an integral which would correctly calculate the desired quantity, but **DO NOT** evaluate it.

(i) (12 points) Arc length of the curve $C = \left\{ \begin{bmatrix} t \\ t^2 \\ t^3 \end{bmatrix} : 0 \leq t \leq 1 \right\}$

(ii) (13 points) Surface area for the part of the paraboloid

$$z = 9 - (x^2 + y^2)$$

lying above the xy-plane, that is, where $z \geq 0$.
(Hint: sketch that part of the paraboloid first!)

3. Prove or disprove in each case.
(i) (6 points) Simpson's numerical approximation using 100 subintervals for the integral $\int_0^1 (x^3+2)dx$ will have value $\frac{9}{4}$.

(ii) (6 points) The indicator function $f(x) = 1_{\mathbb{Q}}(x)$ for the rational numbers inside $\mathbb{R}^1$ is Lebesgue-integrable, with Lebesgue integral $\int_{\mathbb{R}} f(x)|d^1x| = 0$.

(iii) (6 points) The subset $A := [0,1] - \mathbb{Q}$, that is, the *irrational* numbers in the interval $[0,1]$, has measure zero.

(iv) (7 points) This function $\mathbb{R}^1 \xrightarrow{f} \mathbb{R}$ is Riemann-integrable:

$$f(x) = \begin{cases} x^2 & \text{for } x \in \mathbb{Q} \cap [0,1], \\ 0 & \text{otherwise.} \end{cases}$$

4. (25 points) Prove that when n is *odd*, then every $n \times n$ matrix A which is *antisymmetric*, meaning $A^\top = -A$, will have $\det(A) = 0$.

Partial credit given for only verifying the special cases $n = 1$ and $n = 3$.