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To avoid confusion, change “having the nonzero pivots on the diagonal” to “with nonzero diagonal entries”.

Insert “with nonzero diagonal entries” after “diagonal”.

Replace the summand \( j \) by \( j - 1 \):

\[
\sum_{j=1}^{n} (j - 1) = \frac{n^2 - n}{2}
\]

Replace 3210 by 32100:

\[
10x + 1600y = 32100, \quad x + .6y = 22,
\]
First displayed equation:

Replace 3210 by 32100:

\[
\begin{pmatrix}
1600 & 10 & 32100 \\
.6 & 1 & 22
\end{pmatrix}
\]

Exercise 2.3.39 (b):

Add closing bracket to \( W[f(x), g(x)] \).

Exercise 2.4.24 (b):

Change “Under the hypotheses of part (b)” to “Under the hypothesis of part (a)”.

Exercise 2.4.24 (b):

Change “Solving the homogeneous system \( \tilde{U}\mathbf{y} = \mathbf{0} \), we conclude that” to

The two nonzero rows of \( \tilde{U} \) form a basis for corng \( A^T \), and therefore

Exercise 3.2.31 (a):

Add \( T \) superscripts to \( (1, 2, 3)^T \) and \( (1, -1, 2)^T \).

Formula before Proposition 3.34:

Change \( dt \) to \( dx \).

Two lines before Proposition 3.34:

Change Theorem 3.31 to Theorem 3.28.

Exercise 3.4.22 (c):

Change “null vectors” to “null directions”.

Exercise 3.4.32:

Change “null vector” to “null direction” and \( K = A^TA \) to \( K = A^TC A \):

Show that \( \mathbf{0} \neq \mathbf{z} \) is a null direction for the quadratic form \( q(\mathbf{x}) = \mathbf{x}^T K \mathbf{x} \) based on the Gram matrix \( K = A^TC A \) if and only if \( \mathbf{z} \in \ker K \).
Exercise 3.4.35 (c):

Rephrase for clarity:

Show that $K$ is also a Gram matrix, by finding a matrix $A$ such that $K = A^T C A$.

Exercise 3.6.29:

Delete part (e). ("Orthogonal" and "orthonormal" are not yet defined.)

Exercise 3.6.51:

For each of the following . . .

Formula in middle of page:

$y^*$ and $z^*$ should not be bold face.

Theorem 4.4:

Delete the words “null vector”.

Equation (4.26):

The last equality, $c = \| b \|^2$, is correct provided one uses the weighted norm. However, to avoid confusion with the Euclidean norm used in (4.25), it would be better to write this as $c = b^T C b$.

Equation (4.30):

$$\| A x^* - b \| = \sqrt{\| b \|^2 - f^T x^*} = \sqrt{\| b \|^2 - b^T A (A^T A)^{-1} A^T b}.$$
First line under Figure:

Change “x and y” to “v and w”.

Remark:

... solving the homogeneous adjoint system, ...

Exercise 5.6.20 (c):

Change the sign in front of 4$x_3$ in last equation:

$x_1 + 2x_2 + 3x_3 = b_1,$  $x_2 + 2x_3 = b_2,$  $3x_1 + 5x_2 + 7x_3 = b_3,$  $-2x_1 + x_2 + 4x_3 = b_4$;

Equation (5.90):

Change $e^{ikx_n}$ to $e^{ikx_{n-1}}$ in first line.

Line -5:

Change $n = 2^8 = 256$ to $n = 2^9 = 512$.

Equation (6.9):

Insert space between 1 and $-1$ in last row of matrix.

Two lines before (6.15):

Change $Kx = f$ to $Ku = f$.

Four lines after (6.15):

Change $y = A^{-1}f$ to $y = A^{-T}f$.

Exercise 6.2.1 (b):

Change last row of matrix:

$$
\begin{pmatrix}
0 & 0 & 1 & -1 \\
1 & 0 & 0 & -1 \\
0 & -1 & 1 & 0 \\
1 & 0 & -1 & 0
\end{pmatrix}
$$

Exercise 6.2.2:

Add labels to the wires:
Exercise 6.2.12 (a):

Start the exercise with:

Assuming all wires have unit resistance, find the voltage...

Page 313

Line 19:

Delete “the” before “Section 6.1”.

Page 317

Displayed equation above (6.51):

Change 0 to 0.

Page 319

Equation (6.58):

Change the last formula to

\[ z_3 \cdot f = -\frac{\sqrt{3}}{2} f_1 + \frac{1}{2} g_1 + g_2 = 0. \]

Page 319

Last line:

Change “first node” to “third node”.

Page 320

Two lines before displayed equation for \( A^{**} \):

This serves to also eliminate ...

Page 321

Figure 6.13:

Label the bars in the figure:

Page 323

Figure 6.16:

Label the bars in the figure:
Page 324

Figure 6.17:

Label the bars in the figure:

Page 325

Line -5:
Change “three bars” to “five bars”.

Page 339

Equation (7.12):
Add period at end of equation.

Page 384

Exercise 7.5.1 (b):
\( \langle \mathbf{v}, \mathbf{w} \rangle = 2v_1 w_1 + 3v_2 w_2 \)

Page 399

Final displayed equation:
The bar on the second term should extend over both \( A \) and \( \mathbf{v} \):
\[ \overline{A} \mathbf{v} = \overline{A} \mathbf{v} = \overline{\lambda} \mathbf{v} = \overline{\lambda} \mathbf{v}. \]

Page 400

Two lines before Remark:
Change “combinations of the real eigenvalues” to “combinations of the real eigenvectors”.

Page 411

Last line:
Delete “the” before “Section 8.6”.

Page 432

Fourth displayed formula. Switch \( T \) superscript:
\[ A^+ = Q \Sigma^{-1} P^T = \begin{pmatrix} .2444 & .1333 & .0556 & .1889 \\ .1556 & -.0667 & .1111 & .0444 \\ -.1111 & 0 & -.0556 & -.0556 \end{pmatrix}, \]

Page 572

Displayed formula before (10.102):
The subscripts on \( R \) and \( Q \) are wrong:
\[ A_2 = R_1 Q_1. \]
Change equations (10.106) and (10.107) to:

\[ A^k = (Q_0Q_1\cdots Q_{k-1})(R_{k-1}\cdots R_1R_0). \]  \hspace{1cm} (10.106)

\[ S_k = Q_0Q_1\cdots Q_{k-1} = S_{k-1}Q_{k-1}, \] \hspace{1cm} (10.107)

\[ P_k = R_{k-1}\cdots R_2R_1 = R_{k-1}P_{k-1}. \]

Replace the paragraph after Theorem 10.57 by the following:

The last remaining item is a proof of Lemma 10.56. We write

\[ S = (u_1 u_2 \ldots u_n), \quad S_k = (u_{1}^{(k)}, \ldots, u_{n}^{(k)}) \]

in columnar form. Let \(t_{ij}^{(k)}\) denote the entries of the positive upper triangular matrix \(T_k\). The first column of the limiting equation \(S_k T_k \rightarrow S\) reads

\[ t_{11}^{(k)} u_{1}^{(k)} \rightarrow u_{1}. \]

Since both \(u_{1}^{(k)}\) and \(u_1\) are unit vectors, and \(t_{11}^{(k)} > 0\),

\[ \| t_{11}^{(k)} u_{1}^{(k)} \| = t_{11}^{(k)} \rightarrow \| u_{1} \| = 1, \quad \text{and hence} \quad u_{1}^{(k)} \rightarrow u_{1}. \]

The second column reads

\[ t_{12}^{(k)} u_{1}^{(k)} + t_{22}^{(k)} u_{2}^{(k)} \rightarrow u_{2}. \]

Taking the inner product with \(u_{1}^{(k)} \rightarrow u_1\) and using orthonormality, we deduce \(t_{12}^{(k)} \rightarrow 0\), and so \(t_{22}^{(k)} u_{2}^{(k)} \rightarrow u_2\), which, by the previous reasoning, implies \(t_{22}^{(k)} \rightarrow 1\) and \(u_{2}^{(k)} \rightarrow u_{2}\). The proof is completed by working in order through the remaining columns, employing a similar argument at each step. Details are left to the interested reader.

Equation (11.21):

Insert minus sign before integral:

\[ u'(\ell) = - \int_{0}^{\ell} f(x) \, dx = 0, \]  \hspace{1cm} (11.21)

Line before (11.28):

Change “to satisfy” to “satisfy”.

3 lines after (11.40):

Change \(L[u] = u(y)\) to \(L_y[u] = u(y)\).

Equation (11.60):

Missing factor of \(c\) in differential equation:

\[ -cu'' = f(x), \quad u(0) = 0 = u(1), \]
Two missing factors of $c$:

$$
c \frac{du}{dx} = (1 - x) x f(x) + \int_0^x [-y f(y)] dy - x(1-x) f(x) + \int_x^1 (1-y) f(y) dy
$$

$$
= - \int_0^1 y f(y) dy + \int_x^1 f(y) dy.
$$

Differentiating again, we conclude that $c \frac{d^2 u}{dx^2} = -f(x)$, as claimed.

Exercise 11.3.16 (b):
Delete “is” after $K = L^* \circ L$.

Section 11.6, middle of second paragraph:
Change “Chapter 4.1” to “Section 4.1”.

Solution 1.2.4 (d):

$$
A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix}, \quad x = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 1 \end{pmatrix};
$$

Solution 1.2.4 (f):

$$
b = \begin{pmatrix} -3 \\ -5 \\ 2 \\ 1 \end{pmatrix}.
$$

Solution 1.4.15 (a):

$$
\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.
$$

Solution 1.8.4:

(i) $a \neq b$ and $b \neq 0$;  (ii) $a = b \neq 0$, or $a = -2$, $b = 0$;  (iii) $a \neq -2$, $b = 0$.

Solution 1.8.23 (e):

$(0,0,0)^T$;

Solution 2.5.5 (b):

$x^* = (1, -1, 0)^T$,  $z = z \left(-\frac{2}{7}, -\frac{1}{7}, 1\right)^T$;

Solution 3.4.22 (v):

Change “null vectors” to “null directions”.

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Solution 3.4.32:

Change all $x$’s to $z$:

$$0 = z^T K z = z^T A^T C A z = y^T C y,$$
where $y = A z$. Since $C > 0$, this implies $y = 0$, and hence $z \in \ker A = \ker K$.

Solution 4.4.27 (a):

Change “the interpolating polynomial” to “an interpolating polynomial”.

Solution 4.4.52 (b):

Delete the sentence:

(The solution given is for the square $S = \{ 0 \leq x \leq 1, 0 \leq y \leq 1 \}$.)

Solution 5.1.14 (a):

$$v_2 = \pm \left( -\sin \theta, \frac{1}{\sqrt{2}} \cos \theta \right)^T$$

Solution 5.4.15:

$$p_0(x) = 1, \quad p_1(x) = x, \quad p_2(x) = x^2 - \frac{1}{3}, \quad p_3(x) = x^3 - \frac{9}{10} x.$$ (The solution given in the text is for the interval $[0,1]$, not $[-1,1]$.)

Solution 5.5.6 (ii) (c):

$$\left( \frac{23}{43}, \frac{19}{43}, -\frac{1}{43} \right)^T.$$ 

The page layout is a bit strange. The top of the second column (before Solution 6.2.1) is the solution to Exercise 6.1.16(c). Also, the solution to Exercise 6.2.10 spans across both columns.

Solution 6.2.1 (b) The solution corresponds to the revised exercise — see correction on page 311.

For the given matrix, the solution is

Solution 6.2.12 & 6.2.13:

Change all $e$’s to $y$’s.
Solution 6.3.5 (b):

\[
\begin{align*}
\frac{3}{2} u_1 - \frac{1}{2} v_1 - u_2 &= f_1, \\
-\frac{1}{2} u_1 + \frac{3}{2} v_1 &= g_1, \\
- u_1 + \frac{3}{2} u_2 + \frac{1}{2} v_2 &= f_2, \\
\frac{1}{2} u_2 + \frac{3}{2} v_2 &= g_2.
\end{align*}
\]

Solution 8.5.26:

Change (b) to (c).

Solution 11.2.8 (d):

\[
\begin{align*}
f'(x) &= 4 \delta(x + 2) + 4 \delta(x - 2) + \begin{cases} 
1, & |x| > 2, \\
-1, & |x| < 2,
\end{cases} \\
&= 4 \delta(x + 2) + 4 \delta(x - 2) + 1 - 2 \sigma(x + 2) + 2 \sigma(x - 2), \\
f''(x) &= 4 \delta'(x + 2) + 4 \delta'(x - 2) - 2 \delta(x + 2) + 2 \delta(x - 2).
\end{align*}
\]

Solution 11.2.31 (a):

\[
\begin{align*}
u_n(x) &= \begin{cases} 
x(1 - y), & 0 \leq x \leq y - \frac{1}{n}, \\
-\frac{1}{4} n x^2 + \left(\frac{1}{2} n - 1\right) x y - \frac{1}{4} n y^2 + \frac{1}{2} y + \frac{1}{2} x - \frac{1}{4n}, & |x - y| \leq \frac{1}{n}, \\
y(1 - x), & y + \frac{1}{n} \leq x \leq 1.
\end{cases}
\end{align*}
\]

Solution 11.3.3 (c):

(i) \( u_*(x) = \frac{1}{2} x^2 - \frac{5}{2} + x^{-1} \),

(ii) \( P[u] = \int_1^2 \left[ \frac{1}{2} x^2 (u')^2 + 3 x^2 u \right] dx, \quad u'(1) = u(2) = 0 \),

(iii) \( P[u_*] = -\frac{37}{20} = -1.85 \),

(iv) \( P[x^2 - 2] = -\frac{11}{6} = -1.83333, \quad P[-\sin \frac{1}{2} \pi x] = -1.84534 \).

Solution 11.5.7 (b):

\[
\lambda = -\omega^2 < 0, \quad G(x, y) = \begin{cases} 
\frac{\sinh \omega (y - 1) \sinh \omega x}{\omega \sinh \omega}, & x < y, \\
\frac{\sinh \omega (x - 1) \sinh \omega y}{\omega \sinh \omega}, & x > y;
\end{cases}
\]

\[
\lambda = 0, \quad G(x, y) = \begin{cases} 
x(y - 1), & x < y, \\
y(x - 1), & x > y;
\end{cases}
\]

\[
\lambda = \omega^2 \neq n^2 \pi^2 > 0, \quad G(x, y) = \begin{cases} 
\frac{\sin \omega (y - 1) \sin \omega x}{\omega \sin \omega}, & x < y, \\
\frac{\sin \omega (x - 1) \sin \omega y}{\omega \sin \omega}, & x > y.
\end{cases}
\]