MATH 1271: Calculus I

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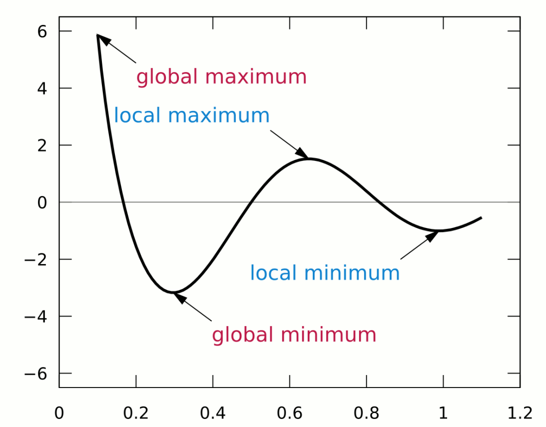
4.1 - Maximum and Minimum Values

Review:



For function f, let c be a number in the domain. We then have...

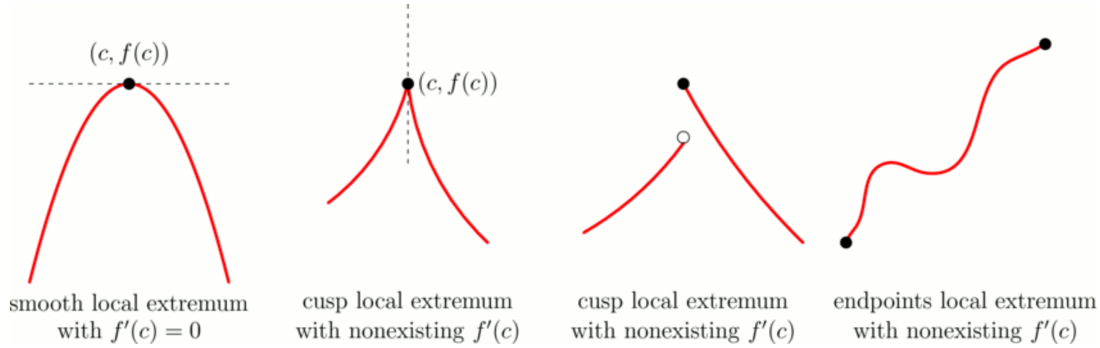
- **Absolute Maximum**: For the value $f(c)$, if $f(c) \geq f(x)$, for all x in the domain.
- **Local Maximum**: For the value $f(c)$, if $f(c) \geq f(x)$, when x near c.

The definition for absolute and local **minimum** follows similarly.

Terminology Note: "value" always refers to the output of the function. So if a problem asks you for the value, don't give the number $x = 2$ from the domain as your answer, give $f(2)$!

Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$, then f has an absolute maximum and absolute minimum on $[a, b]$. Observe that for a constant function, the absolute maximum is also the absolute minimum!

Fermat's Theorem: If f has a local maximum or minimum (extremum) at c, and if the function is differentiable (AKA "smooth") at c, then $f'(c) = 0$.



Critical Number: A number c in the domain such that: either $f'(c) = 0$, or $f'(c)$ does not exist (this obviously includes local maximums and minimums, discontinuities, and endpoints of the domain).

Closed Interval Method (**for finding absolute extrema**): For an interval,

- Find the values of the function at the critical numbers (including endpoints of domain!)

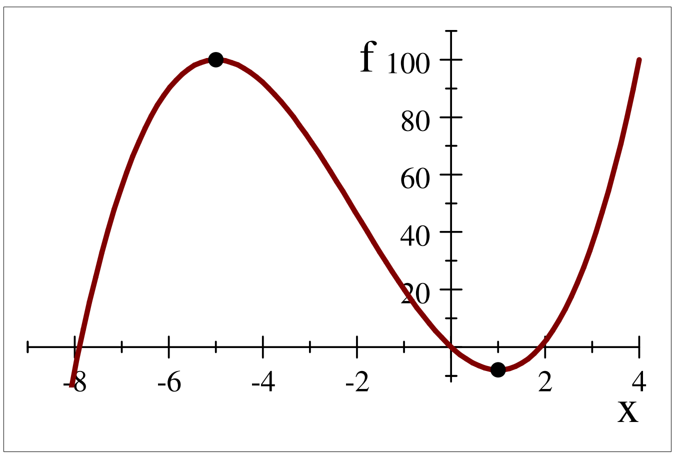
- The largest of the values from the previous two steps is the absolute maximum, and the smallest of these is the absolute minimum.

Problem 30. Find the critical numbers of the function $f(x) = x^3 + 6x^2 - 15x$.

$$f'(x) = 3x^2 + 12x - 15$$

$$= 3(x^2 + 4x - 5) = 3(x+5)(x-1).$$

$f'(x) = 0$ when $x = -5$ or 1. These are the only critical numbers because f', being a polynomial, is defined everywhere.



$x^3 + 6x^2 - 15x$

Problem 54. Find the absolute maximum and absolute minimum values of $f(x) = \frac{x}{x^2-x+1}$ on $[0, 3]$.

$$f'(x) = \frac{(x^2-x+1)-x(2x-1)}{(x^2-x+1)^2} = \frac{x^2-x+1-2x^2+x}{(x^2-x+1)^2} = \frac{1-x^2}{(x^2-x+1)^2}$$

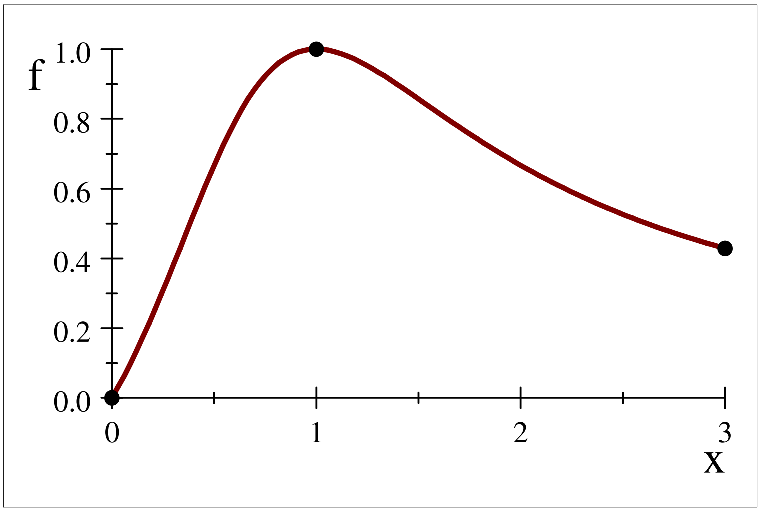
$$= \frac{(1+x)(1-x)}{(x^2-x+1)^2} = 0.$$

From this we find critical values (when numerator is zero) of $x = \pm 1$.

However, $x = -1$ is not in the given interval, $[0, 3]$.

Checking the **values** of the critical points and boundary points:
$f(0) = 0,\ f(1) = 1$, and $f(3) = \frac{3}{7}$.

So $f(0) = 0$ is the absolute minimum value and $f(1) = 1$ is the absolute maximum value.



$\frac{x}{x^2-x+1}$

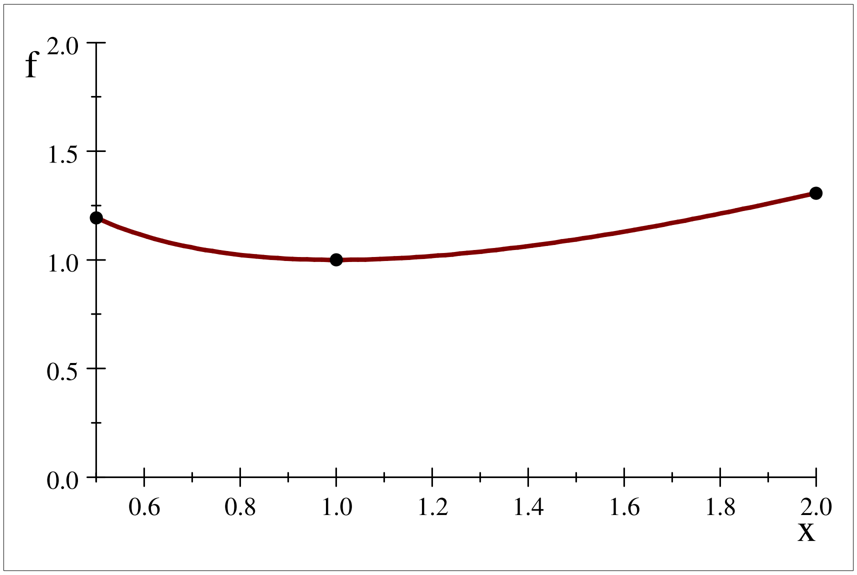
Problem 60. Find the absolute maximum and absolute minimum values of
$f(x) = x - \ln x$ on $[\frac{1}{2}, 2]$.

$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$.

$f'(x) = 0$ when $x = 1$.

Checking the critical points and boundary points: $f(\frac{1}{2}) = \frac{1}{2} - \ln\frac{1}{2} \approx 1.19$, $f(1) = 1$, and $f(2) = 2 - \ln 2 \approx 1.31$.

So $f(2) = 2 - \ln 2$ is the absolute maximum value and $f(1) = 1$ is the absolute minimum value.



$x - \ln x$