MATH 1271: Calculus I

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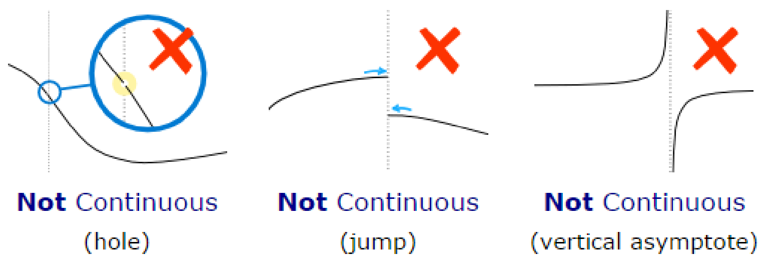
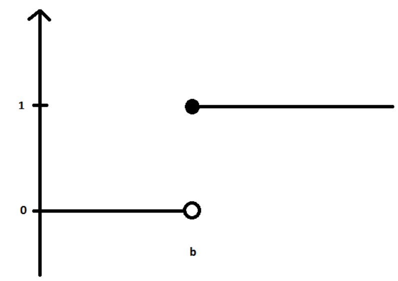
2.5 - Continuity

Review:





Continuous from the right at b

A function f is continuous: ♦ At a number a if $\lim_{x\to a} f(x) = f(a)$.

♦ From the **right** at a number a if $\lim_{x\to a^+} f(x) = f(a)$.

♦ From the **left** at a number a if $\lim_{x\to a^-} f(x) = f(a)$.

Definition: A function f is **continuous on an interval** if it is continuous at every number in the interval. (If f is defined at an end point of the interval, we understand continuous at that end point to mean continuous from the right/left).

Stability of Continuity over Operations: If f and g are continuous at a, and c is a constant, then the following functions are also continuous at a :

♦ $f+g$, ♦ $f-g$, ♦ cf, ♦ fg, ♦ $\frac{f}{g}$ if $g(a) \neq 0$.

Continuity of Polynomials and Rational Functions:

♦ Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.

♦ Any rational function is continuous whenever it is defined; that is, it is continuous on its domain (for example $\frac{1}{x+5}$ is continuous everywhere except $x = -5$).

Functions that are continuous at every number in their domains:
Polynomials, Rational Functions, Root Functions, Trigonometric Functions, Inverse Trigonometric Functions, Exponential Functions, Logarithmic Functions.

Continuity of Function Composition:

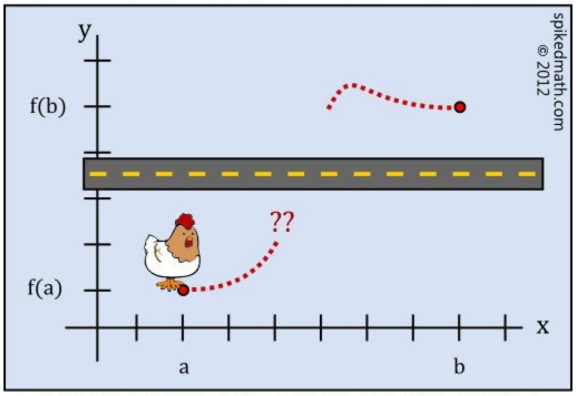
♦ If f is continuous at b and $\lim_{x\to a} g(x) = b$, then $\lim_{x\to a} f(g(x)) = f(b)$.

In other words, $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$.

♦ If g is continuous at a and f is continuous at $g(a)$, then the composition function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

The Intermediate Value Theorem: Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number of c in (a, b) such that $f(c) = N$.

WHY DID THE CHICKEN CROSS THE ROAD?



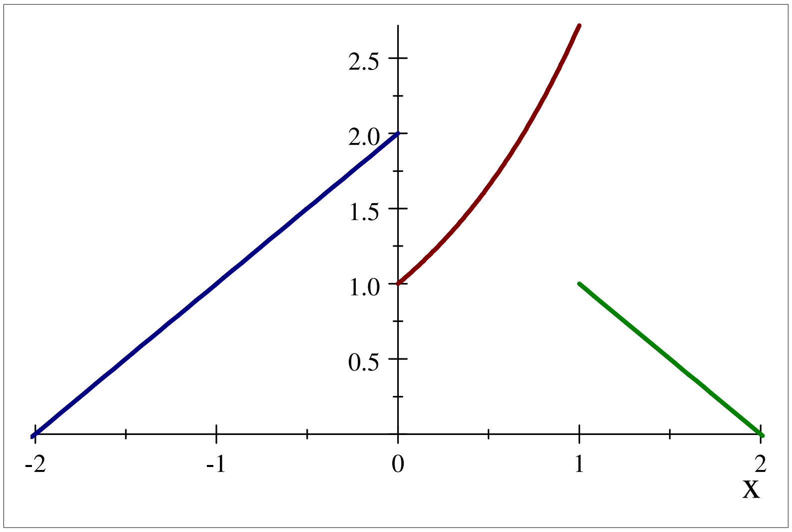
THE INTERMEDIATE VALUE THEOREM.

Problem 6. Sketch the graph of a function f that is continuous except for discontinuities at -1 and 4, but is continuous from the left at -1 and from the right at 4.

Problem 8. Sketch the graph of a function f that is neither left nor right continuous at -2, and continuous only from the left at 2.

Problem 43. Where is f continuous from the right, from the left, or neither? Sketch the graph of f.

43. $$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^x & \text{if } 0 \leqslant x \leqslant 1 \\ 2 - x & \text{if } x > 1 \end{cases}$$



Problem 49. If $f(x) = x^2 + 10\sin x,$ show that there is a number c such that $f(c) = 1000.$

We know that x^2 is continuous,

and similarly that $10\sin x$ is continuous.

Furthermore, we know that the sum of two continuous functions is also continuous.

$f(0) = 0^2 + 10\sin(0) = 0 + 0 = 0.$

$f(100) = 100^2 + 10\sin(100) \geq 100,00 + 10(-1) = 9,990.$ (since $1 \geq \sin(x) \geq -1$)

By the intermediate value theorem, we know that there must be a $c \in (0, 100)$ such that $f(c) = 1000.$

Problem 53. Use the intermediate value theorem to show that there is a root of the given equation in the specified interval.
$e^x = 3 - 2x,\ (0, 1)$

Let's call $f(x) := e^x + 2x - 3.$ Does $f(x) = 0$ on $(0, 1)$?

This function is continuous on $[0, 1]$

$f(0) = -2$ and

$f(1) = e - 1 \approx 1.72.$

Since $-2 < 0 < e - 1$, there exists a number $c \in (0, 1)$ such that $f(c) = 0$ by IVT.
So, there is a root to our equation in the specified range.