MATH 1272: Calculus II

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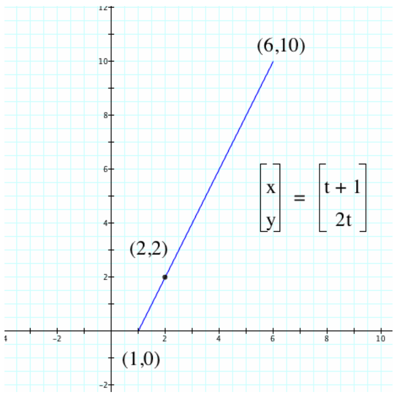
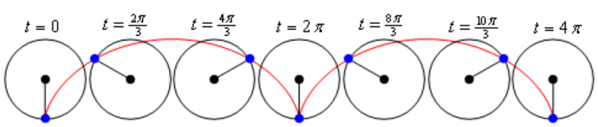
10.1 Curves Defined by Parametric Equations

Review:

Defining a curve in two dimensions.



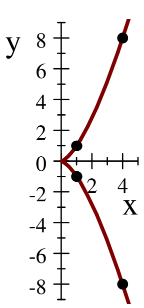
If the coordinates of the points in the curve are defined by functions $x = f(t)$ and $y = g(t)$ where $-\infty < t < \infty$, we say that this curve is **parameterized** by t, and the equations above are called **parametric equations**. Observe that for every t, we have a point $(x, y) = \big(f(t),\ g(t)\big)$ on the **parametric curve**.



The Cycloid

Imagine a circle sitting on the origin such that the point P at the bottom of the circle is currently located at $(0,0)$. However, now imagine that you roll the circle (as you would a tire down the street) along the x-axis. Now imagine the path that point P makes as it leaves the origin and travels along with the rolling circle. This path (seen above) is called a **cycloid**, and the examination of this shape has applications including celestial mechanics and the building of bridges.

Problem #10 a) Sketch the curve $x = t^2, \quad y = t^3$ by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.



t	-2	-1	0	1	2
x	4	1	0	1	4
y	-8	-1	0	1	8

b) Eliminate the parameter to find a Cartesian equation of the curve.

$y = t^3 \quad \Rightarrow \quad t = \sqrt[3]{y}$

$$x = t^2 = \left(\sqrt[3]{y}\right)^2 = y^{\frac{2}{3}}, \text{ where } t \in \mathbb{R},\ y \in \mathbb{R},\ x \geq 0.$$

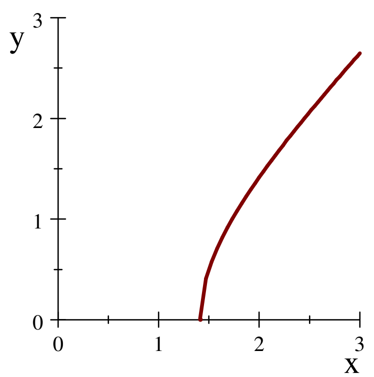
Problem #16 a) Eliminate the parameter to find a Cartesian equation of the curve $x = \sqrt{t+1}, \quad y = \sqrt{t-1}$.

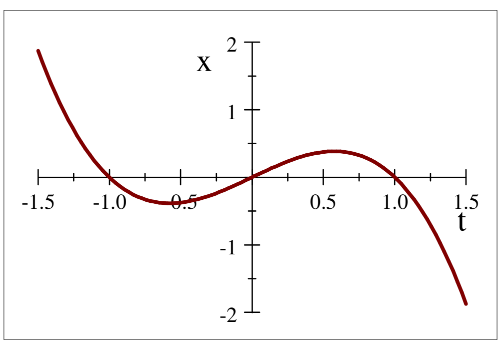
$x^2 = t + 1 \quad \Rightarrow \quad t = x^2 - 1.$

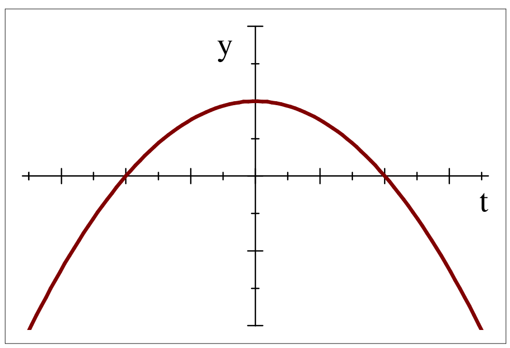
$y = \sqrt{t-1} = \sqrt{(x^2-1)-1} = \sqrt{x^2-2}.$

The curve is the part of the hyperbola: $x^2 - y^2 = 2$, with $x \geq \sqrt{2}$ and $y \geq 0$.

b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.







Problem #26 Use the graph of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t),\ y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.

For $t < -1$, x is positive and decreasing, while y is negative and increasing (these points are in quadrant IV).

When $t = -1$, $(x, y) = (0, 0)$ and, as t increases from -1 to 0, x becomes negative and y increases from 0 to 1.

At $t = 0$, $(x, y) = (0, 1)$ and, as t increases from 0 to 1, y decreases from 1 to 0 and x is positive.

At $t = 1$, $(x, y) = (0, 0)$ again, so the loop is completed.

For $t > 1$, x and y both become large negative. This enables us to draw a rough sketch. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.