Set 1

1. Venus and Mars

a. Compute the solar flux in Watts per square meter for Venus and Mars. You may assume that the orbit of Venus is at a distance of 1.08×10^{11} meters from the Sun while Mars is at a distance of 2.28×10^{11} meters.

b. Assume that Venus and Mars are replaced in their orbits by perfect black bodies. What would their surface temperatures be?

c. Assume that Venus is replaced by an otherwise perfect black body, but with an albedo of 0.71. What would the surface temperature be? The actual albedo of Venus is about 0.71, and the surface temperature is approximately 737K. Discuss any discrepancy between this value and your computed value.

d. Assume that Mars is replaced by an otherwise perfect black body, but with an albedo of 0.17. What would the surface temperature be? The actual albedo of Mars is about 0.17, and the surface temperature is approximately 213K. Discuss any discrepancy between this value and your computed value.

Solution. Recall the formulas from class (9/7/2023).

The solar flux F at a distance r from the sun is given by

$$F = 6.33\times10^{7}\left(\frac{r_S}{r}\right)^2 \text{ W/m}^2,$$

where $6.33\times10^{7}\text{ W/m}^2$ is the solar flux at the surface of the sun, and where $r_S = 6.96\times10^{8}\text{ m}$ is the solar radius.

The surface temperature of a black body is given by

$$Q = \sigma T^4,$$

where $Q = F/4$ is the average planetary surface flux, $\sigma = 5.67\times10^{-8}\text{ W/m}^2K^4$ is the Stefan-Boltzmann constant, and T is the surface temperature in Kelvin. Therefore,

$$T = \left(Q/\sigma\right)^{1/4}.$$

If the planet has an albedo of α, then the surface temperature becomes

$$T = \left((1-\alpha)Q/\sigma\right)^{1/4}.$$

a. For Venus, $r = 1.08\times10^{11}\text{ m}$, so the flux is $F = \boxed{2629\text{ W/m}^2}$.

For Mars, $r = 2.28\times10^{11}\text{ m}$, so the flux is $F = \boxed{590\text{ W/m}^2}$.

b. For Venus, $Q = F/4 = 657$, so the surface temperature would be $T = (Q/\sigma)^{1/4} = \boxed{328\text{ K}}$, ($55\ ^\circ\text{C}$ or $131\ ^\circ\text{F}$).

For Mars, $Q = F/4 = 147$, so the surface temperature would be $T = (Q/\sigma)^{1/4} = \boxed{226\ \mathrm{K}}$, ($-47\ ^\circ\mathrm{C}$ or $-53\ ^\circ\mathrm{F}$).

c. For Venus, $T = \left((1-\alpha)Q/\sigma\right)^{1/4} = \left(0.29 \times 657/\sigma\right)^{1/4} = \boxed{241\ \mathrm{K}}$, ($-32\ ^\circ\mathrm{C}$ or $-26\ ^\circ\mathrm{F}$). Venus has a very thick atmosphere, consisting mostly of carbon dioxide, giving a "runaway greenhouse" effect and allowing the surface temperature to reach $737\ \mathrm{K}$ ($464\ ^\circ\mathrm{C}$ or $867\ ^\circ\mathrm{F}$).

d. For Mars, $T = \left((1-\alpha)Q/\sigma\right)^{1/4} = \left(0.83 \times 147/\sigma\right)^{1/4} = \boxed{216\ \mathrm{K}}$ ($-58\ ^\circ\mathrm{C}$ or $-72\ ^\circ\mathrm{F}$). Mars has a very thin atmosphere with an insignificant greenhouse effect. The actual surface temperature of $213\ \mathrm{K}$ is consistent with this model.