Homework Set 4

Due October 19, 2023

Please refer to Table S1 in the 2005 Hansen *Science* paper (James Hansen, et al, *Earth's Energy Imbalance: Confirmation and Implications*, SCIENCE **308** (2005), p. 1431), repeated here for your convenience.

Table S1. Planetary Heat Storage: Ocean, Ice, Air and Land.

Energy required to melt ice and warm the air, land and ocean by specified amounts.[1]

Ocean warming by 1°C through 1 km depth of ocean. Heat storage is 1°C × 10^5 g/cm^2 × 1 cal/g × 4.19 joules/cal × area Earth × 0.7 ~ 15 × 10^{23} joules ~ **93 W yr/m^2**.

Ice sheet melting to raise sea level 1 meter. Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C). Energy required is 100 cal/g (80 cal/g for melting). Energy for 1 meter of sea level: 100g/cm^2 × 100cal/g × 4.19 joules/cal × area Earth × 0.7 ~1.5 × 10^{23} joules ~ **9.3 W yr/m^2**.

Sea ice melting (all sea ice on planet). Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C), and that sea ice covers 4% of the planet with mean thickness 2.5 m. Energy required is 250 g cm^2 × 100 cal/g (80 cal/g for melting) × 4.19 joules/cal × 0.04 × area Earth ~ 2.14 × 10^{22} joules ~ **1.3 W yr/m^2**.

Air warming by 1°C. The Earth's atmospheric mass is ~ 10 m of water. Heat capacity of air ~ 0.24 cal/g/°C. Energy to raise air temperature 1°C: 1°C × 1000 g/cm^2 × 0.24 cal/g/°C × 4.19 joules/cal × area Earth ~ 0.26 ×10^{22} joules ~ **0.32 W yr/m^2.**

Land surface warming by 1°C: The depth of penetration of a thermal wave into the Earth's crust in 10 years, weighted by ΔT, is ~10 m. With density ~ 3 g/cm^3, heat capacity ~ 0.2 cal/g/°C, and 0.29 fractional land coverage, land heat storage is 10^3 cm ×3 g/cm^3 × 0.2 cal/g/°C × 1°C ×4.19 joules/cal × area Earth × 0.29 ~ 0.37 × 10^{22} joules ~ **0.23 W yr**. [In a century the depth of penetration is ~3 times more than in a decade, so heat storage in a century due to 1°C warming is ~ **0.7 W yr/m^2**.]

[1]Note that 1 W sec = 1 joule, # sec/year ~ π×10^7, area Earth ~ 5.1×10^{18} cm^2, 1 W yr over full Earth ~ 1.61×10^{22} joules, ocean fraction of Earth ~ 0.7, 1 calorie ~ 4.19 joules.

1. Assuming an energy imbalance of 1.2 W/m^2, how long would it take to raise the temperature of the entire ocean by 5°C? (You may assume that the average ocean depth is 4300 meters). How long would it take to melt enough ice to raise the ocean depth by 70 meters, leaving the ocean temperature unchanged?

2. By some estimates, it took 20,000 years to raise the temperature of the entire ocean 5°C during the PETM. What energy imbalance would account for that rise? (Assume all other temperatures were unchanged and that the ocean depth was 4400 meters.)

3. What energy imbalance would be required to melt enough ice to raise the ocean depth by 70 meters in 1000 years, assuming all atmosphere, land, and ocean temperatures remained constant? What energy imbalance would accomplish the same rise in 100 years?

4. Assume that, over the course of 100,000 years, the air and land temperatures fell by 5°C, as did the top kilometer of the ocean, and enough glaciers formed to lower the sea level by 125 meters. What average energy imbalance would be required?

5. (*The Day After Tomorrow* scenario). Assume that, over the course of 6 weeks, the air and land temperature dropped by 10°C, as did the top 100 meters of the ocean, and enough snow accumulated on land to lower the sea level by 2 meters. What energy imbalance would be required?