Permafrost Response to Climate Change via Budyko's Model

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Mathematics of Climate Seminar
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Mathematics and Climate Research Network

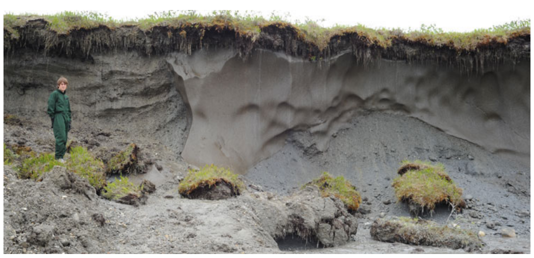
Permafrost Response

Suppose that we are able to hold the increase in global mean temperature to 2°C. What will happen to the permafrost? Can an energy balance model tell us anything?

https://climatekids.nasa.gov/permafrost/

Permafrost Response

What is permafrost?



https://www.nps.gov/gaar/learn/nature/permafrost.htm

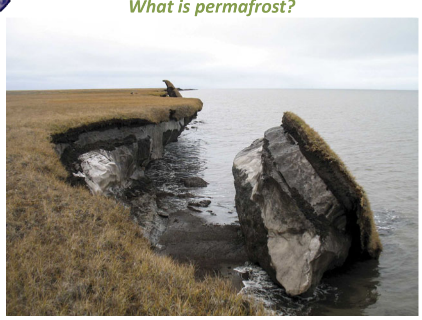
Permafrost Response

What is permafrost?

https://climatevulture.com/2016/08/28/satellite-remote-sensing-of-permafrost/

Permafrost Response

What is permafrost?



http://alaska.usgs.gov/science/interdisciplinary_science/cae/arctic_coastal_plain.php

Permafrost Response

What is permafrost?

Washington Post, Oct 4, 2019: "In fast-thawing Siberia, radical warming is warping the earth"

Permafrost Response

Where is the permafrost?

Average latitude of permafrost boundary: 61°

(yellow circle)

(Aileen Zebrowski)

Permafrost Response

Potential Carbon Emissions

The National Snow and Ice Data Center estimates that there are *1400* Gigatonnes of carbon (GtC) stored in the permafrost.
https://nsidc.org/cryosphere/frozenground/methane.html

By comparison, the atmosphere currently holds about *890* GtC.

Metric Conversions
1 tonne = 10^3 kilograms = 10^6 grams = 1 megagram
1 gigatonne (Gt) = 10^9 tonnes = 10^{12} kilograms = 10^{15} grams = 1 petagram (Pg)

Carbon Conversions
atomic weights: Carbon: 12, Oxygen: 16
molecular weight of CO_2: 44
ppm = parts per million in the atmosphere
GtC = Gigatonnes of Carbon
1ppm CO_2 = 2.13 GtC = 7.81 $GtCO_2$
420 ppm CO_2 = 895 GtC = 3280 $GtCO_2$

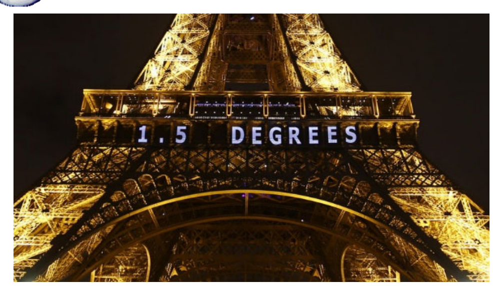
Permafrost Response

2015 Paris climate conference (COP21)

http://www.cbc.ca/radio/thecurrent/a-special-edition-of-the-current-for-november-30-2-degrees-1.3343179

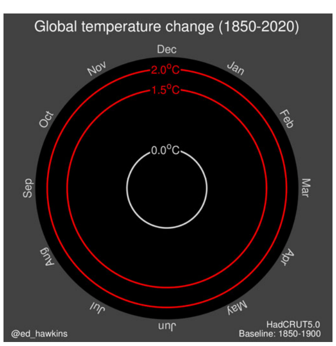
Permafrost Response

2015 Paris climate conference (COP21)



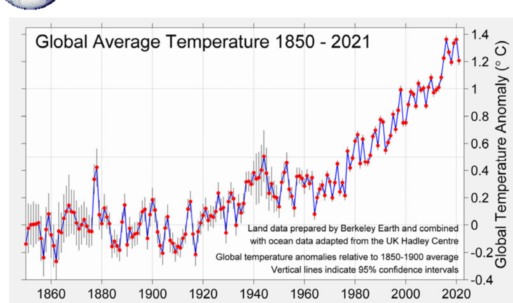
http://www.npr.org/sections/thetwo-way/2015/12/12/459502597/2-degrees-100-billion-the-world-climate-agreement-by-the-numbers

Permafrost Response



https://www.climate-lab-book.ac.uk/spirals/

Permafrost Response



http://berkeleyearth.org/global-temperature-report-for-2021/

Permafrost Response

How much carbon would be released from the permafrost if the global mean temperature rose to 2 degrees Celsius above pre-industrial levels?

Permafrost Response

Energy Balance

temperature change ~ energy in – energy out

short wave energy from the Sun

long wave energy from the Earth

Everything else is detail.

Permafrost Response

Budyko's Equation

surface temperature

sin(latitude)

$\bar{T} = \int_0^1 T(y)dy$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\bar{T}-T)$$

heat capacity

insolation

albedo

OLR

heat transport

Symmetry assumption: $0 \le y = \sin(\text{latitude}) \le 1$

Permafrost Response

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\bar{T}-T)$$

parameter	value	units
Q	343	Wm^{-2}
s_2	−2.41	dimensionless
A	202	Wm^{-2}
B	1.9	$\mathrm{Wm}^{-2}\mathrm{K}^{-1}$
C	3.04	$\mathrm{Wm}^{-2}\mathrm{K}^{-1}$
α_1	0.32	dimensionless
α_2	0.62	dimensionless
T_c	−10	$^\circ\mathrm{C}$

K.K. Tung, *Topics in Mathematical Modeling*, Princeton University Press, 2007, Chapter 8.

Permafrost Response

Budyko's Equation

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→ Chylek and Coakley's quadratic approximation:

→ $s(y) \approx 1 + s_2(3y^2-1)$, where $s_2 = -0.241$

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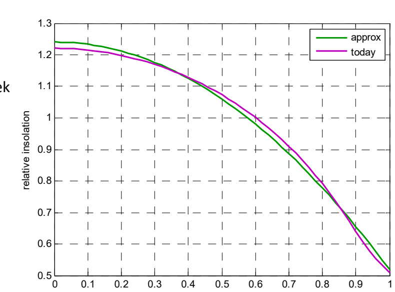
Insolation Distribution

green = quadratic approximation (Chylek & Coakley)

fuchsia = formula using obliquity of 23.4°



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Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\overline{T}-T)$$

surface temperature; sin(latitude); $\overline{T} = \int_0^1 T(y)dy$

heat capacity; insolation; **albedo**; OLR; heat transport

$$\alpha(y) = \begin{cases} \alpha_1 = 0.32, & y < \eta, \\ \alpha_2 = 0.62, & y > \eta, \end{cases}$$

current ice boundary

Permafrost Response

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\overline{T}-T)$$

surface temperature; sin(latitude); $\overline{T} = \int_0^1 T(y)dy$

heat capacity; insolation; albedo; OLR; **heat transport**

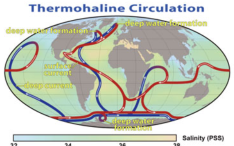
relaxation to mean $C(\overline{T}-T(y))$

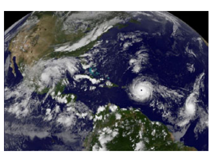
global mean temperature; temperature at latitude y

Permafrost Response

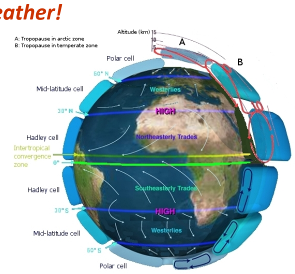
Relaxation to Mean

Thermohaline Circulation





Weather!



Permafrost Response

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\overline{T}-T)$$

surface temperature; sin(latitude); $\overline{T} = \int_0^1 T(y)dy$

heat capacity; insolation; albedo; OLR; heat transport

Equilibrium Solution: $\frac{\partial T}{\partial t} = 0$

Permafrost Response

Equilibrium Temperature Profile

$$T(y) = \frac{1}{B+C}\left(Qs(y)(1-\alpha(y)) - A + C\overline{T}\right)$$

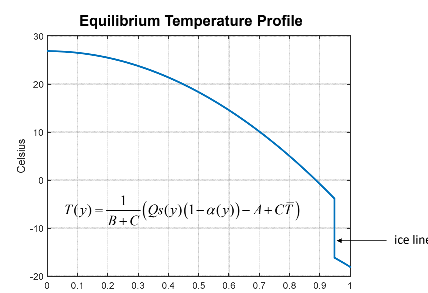
where $\alpha(y) = \begin{cases} \alpha_1 = 0.32, & y < \eta, \\ \alpha_2 = 0.62, & y > \eta, \end{cases}$ current ice boundary

global mean temperature $\longrightarrow$ $\overline{T} = \frac{1}{B}\left(Q(1-\overline{\alpha}) - A\right)$, and $\overline{\alpha} = \int_0^1 \alpha(y)s(y)dy.$ $\longleftarrow$ average albedo

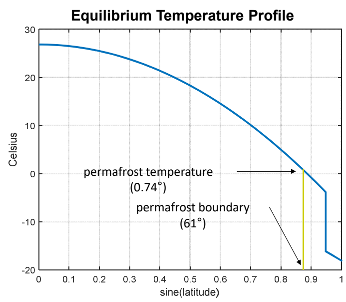
$$T(y) = \begin{cases} \frac{1}{B+C}\left(Qs(y)(1-\alpha_1) - A + C\overline{T}\right), & y < \eta, \\ \frac{1}{B+C}\left(Qs(y)(1-\alpha_2) - A + C\overline{T}\right), & y > \eta. \end{cases}$$

piecewise quadratic

Permafrost Response



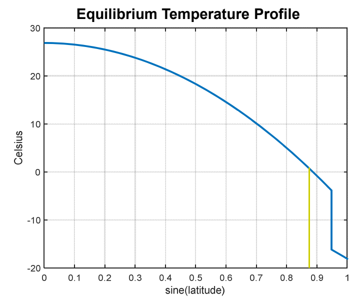
Permafrost Response



Permafrost Response

If we raise the global mean temperature by 2°C, how will the permafrost boundary move?

First approximation: Raise the temperature profile by 2°, and use linear approximation to determine the latitude where the new temperature is the permafrost temperature.



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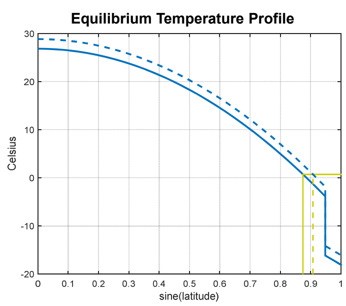

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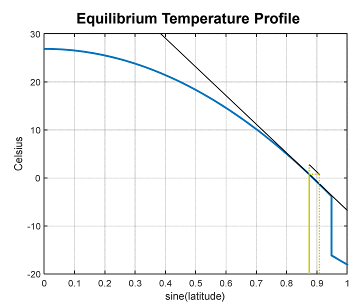

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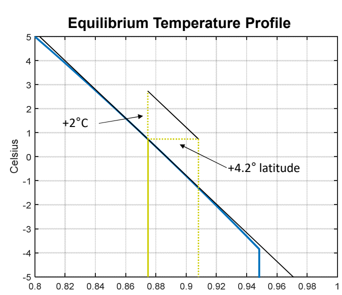
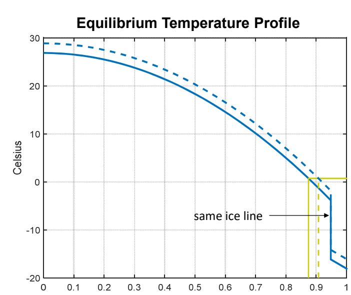


Permafrost Response

If we raise the global mean temperature by 2°C, how will the permafrost boundary move?

We have not taken into account that the ice line might move.



Permafrost Response

Global Mean Temperature

$$\bar{T}(\eta) = \frac{1}{B}\big(Q(1-\bar{\alpha}(\eta)) - A\big), \text{ where } \bar{\alpha}(\eta) = \int_0^1 \alpha(y,\eta)s(y)dy,$$

$$\text{where } \alpha(y) = \begin{cases} \alpha_1 = 0.32, & y < \eta, \\ \alpha_2 = 0.62, & y > \eta, \end{cases} \quad \text{ice line}$$

The ice line is determined by the assumption that the average temperature across the ice line is $\boxed{T_c, \text{ usually taken to be } -10°\text{C.}}$ This condition reduces to

$$\frac{1}{B+C}\big(Qs(\eta)(1-\alpha_0)\big) - A + C\bar{T}(\eta)\big) = T_c, \quad \text{where } \alpha_0 = \frac{1}{2}(\alpha_1 + \alpha_2)$$

outgoing long wave radiation varies with greenhouse gases.

$$h(\eta, A) = \frac{1}{B+C}\left(Qs(\eta)(1-\alpha_0)) - A + \frac{C}{B}\big(Q(1-\bar{\alpha}(\eta)) - A\big)\right) - T_c = 0$$

e.g., McGehee & Widiasih 2014, *SIAM J. Applied Dynamical Systems* 13, pp 518-536.

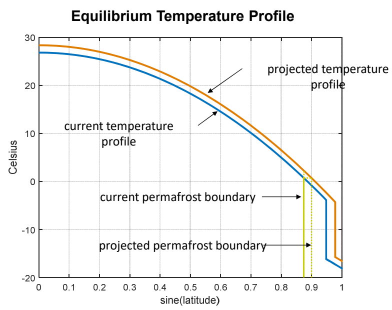
Permafrost Response

How to Proceed?

1. Determine how the ice line varies with the parameter *A*.
2. Determine the change in *A* giving an increase of 2 degrees Celsius in the global mean temperature.
3. Determine the change in the location of the permafrost boundary given the change in *A*.

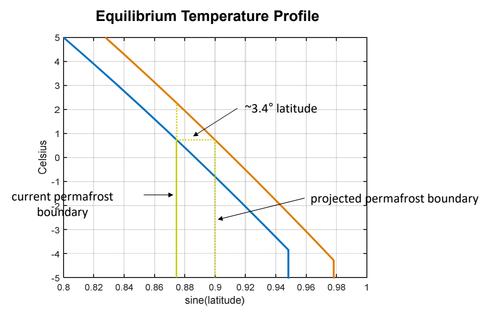
Permafrost Response

Equilibrium Temperature Profile



Permafrost Response

Equilibrium Temperature Profile

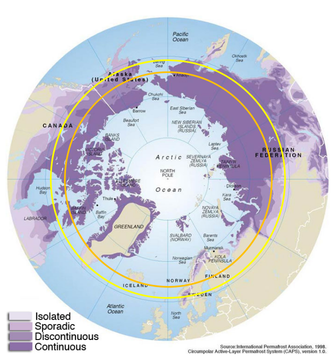


Permafrost Response

Where is the permafrost?

Average latitude of permafrost boundary: 61° (yellow circle)

Projected permafrost boundary (orange circle)



Permafrost Response

How much carbon would be released from the permafrost if the global mean temperature rose by 2 degrees Celsius?

Recall that the surface area is proportional to y, the sine of the latitude.

Current permafrost boundary: $y_p = \sin(61^\circ) \approx 0.875$

Proportion of globe cover by permafrost: $1 - y_p = 0.125$

$\Delta y \approx 0.027$

Proportion of permafrost melted: $\dfrac{0.027}{0.125} = 0.216$

Amount of carbon released: $0.216 \times 1400 = \boxed{302 \text{ GtC}}$

Total fossil fuel emissions since 1751: 375 GtC

To hold the GMT at 2°C, we will have to withdraw 300 GtC from the atmosphere as the permafrost melts.

Permafrost Response

How fast will the permafrost melt?

Can we couple Budyko's equation with a model of permafrost melting to get a simple dynamical model?

Maria Sanchez Muniz

Kaitlin Hill

Permafrost Response

Siberian Permafrost Sinkhole

Permafrost Response

Siberian Permafrost Sinkhole

Permafrost Response

Other Interesting Questions

Budyko's model includes ice-albedo feedback, but not carbon feedback.

- Can we modify the model to include the effects of permafrost melt on atmospheric carbon?
- Could we use the data we have about current permafrost to model the glacial retreats during the Pleistocene?*
- To what extent was the "dead ice" in the Holocene similar to today's permafrost?**

*e.g., J.A. Walsh, E. Widiasih, J. Hahn & R. McGehee, *Nonlinearity* 29, 1843-1864 (2016).

**H. Wright & I. Stefanova, *Acta Palaeobotanica* 44, 141–146 (2004).

Permafrost Response

Holocene Permafrost?

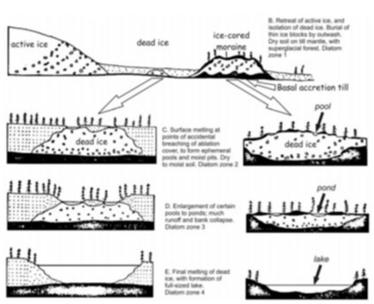


Fig. 2. Sequential diagrams illustrating the formation of trash layers at the base of fine organic lake sediments. From Florin and Wright (1969)

Permafrost Response

http://www.rootsweb.ancestry.com/~ndpembin/html/lake_agassiz.htm

Permafrost Response

Could this be a picture of the North shore of Lake Agassiz 10,000 years ago?

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Permafrost Response

Next Week

Can we couple Budyko's equation with a model of permafrost melting to get a simple dynamical model?

Progress Report

Maria Sanchez Muniz

Kaitlin Hill

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