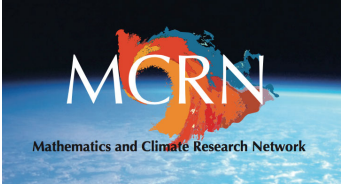


The Climate Emergency A Mathematical Perspective

Richard McGehee
School of Mathematics
University of Minnesota
Mathematics of Climate Seminar
September 6, 2022





The Climate Emergency




Greta Thunberg, Stockholm, August 28, 2018

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The Climate Emergency

Greta Thunberg, Person of the Year 2019

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The Climate Emergency




Greta Thunberg, Person of the Year 2019

People are suffering. People are dying. Entire ecosystems are collapsing. We are in the beginning of a mass extinction, and all you can talk about is money and fairy tales of eternal economic growth.

Greta Thunberg, UN speech, September 23, 2019

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The Climate Emergency



Donald Trump, January 6, 2021

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Greta Thunberg, UN speech, September 23, 2019

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The Climate Emergency

Rey, Star Wars



Emperor Palpatine, Star Wars

I haven't come to lead the Sith. I've come to end them.

Rey, Episode IX – The Rise of Skywalker

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The Climate Emergency Floods

America's summer of floods: climate crisis fueling barrage, scientists say



The Guardian

© Reuters. Flooding in East St. Louis, Illinois, in late July. Photograph: Derek Ross/Reuters


Yellowstone Death Valley: Scientists say experts say extreme rainfall spurred by global heating is rendering historical norms obsolete

www.theguardian.com/us-news/2022/aug/11/america-summer-floods-rainfall-climate-crisis

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The Climate Emergency Droughts

Hoover Dam, USA



THE WALL STREET JOURNAL

Europe's Key Rivers Fall to Critical Levels, Aggravating Energy Crunch

Low water levels and heat throughout Europe and elsewhere have led to an energy emergency due to Russia's gas cut


www.wsj.com/articles/europes-key-rivers-fall-to-critical-levels-aggravating-energy-crisis-1166021654

Loire River, France

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The Climate Emergency Fires

France



Forest fires across the EU

- France**
South western Grande region
- Portugal**
Northern Portugal east to the city of Porto
- Spain**
Southern Spain including Major hills and Pyrenees and north western Zamora region
- Morocco**
Larache, Essaouira, Marrakech and Safi and Agadir provinces
- Greece**
Fires near east Athens and Hellas and Northern Crete
- Italy**
Po Valley


www.nationalworld.com/news/weather/europe-wildfires-2022-map-of-areas-affected-by-land-sliding-france-spain-portugal-how-do-wildfires-start-372847

www.npr.org/2022/07/18/111996473/france-wildfires-heat


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The Climate Emergency Heat Waves

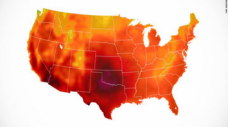
London



Phoenix



USA Heat Map



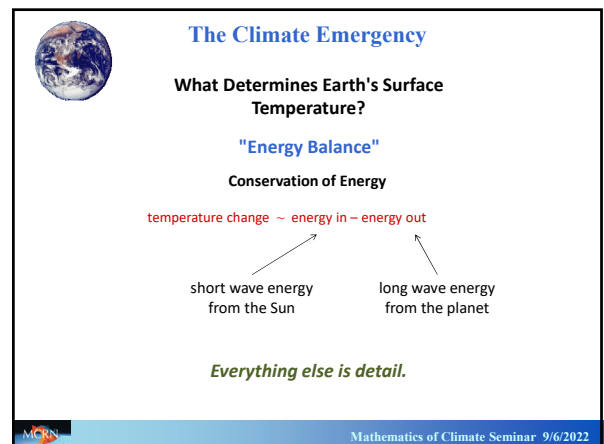
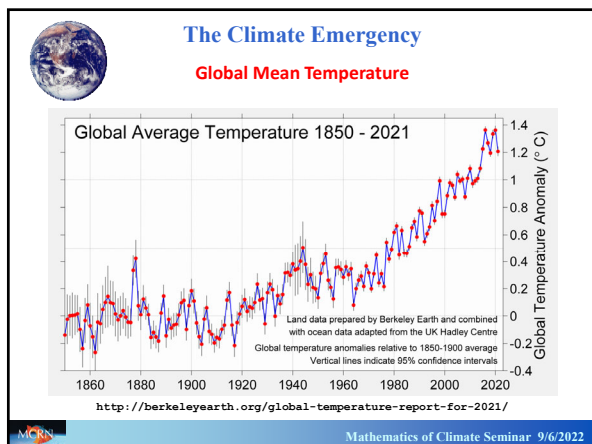
<https://news.un.org/en/story/2022/07/1122822>


abcnews.go.com/Technology/record-breaking-heat-wave-us-europe-prove-climate/story?id=97069737

www.bbcnews.com/4/explains-what-is-behind-heat-wave-affecting-united-states/6871570.html

www.cnn.com/2022/07/19/weather/us-heat-extreme-warnings-forecast/index.html

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The Climate Emergency


Stefan-Boltzmann Law

$$F = \sigma T^4$$


power flux (W/m²) temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

Reasonable approximation:
Every body in the solar system radiates energy according to this law.



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The Climate Emergency



Stefan-Boltzmann Law

$$F = \sigma T^4$$


power flux (W/m²) temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

Example
 surface temperature of the Sun: 5780K
 power flux: $5.67 \times 10^{-8} \times (5780)^4 = 6.33 \times 10^7 \text{ W/m}^2$
 total solar power output: $6.33 \times 10^7 \times 4\pi(r_s)^2$,
 where r_s = radius of the sun = $6.96 \times 10^8 \text{ m}$
 total solar output: $3.85 \times 10^{26} \text{ W}$
 230 nanoseconds = time it takes for the Sun to produce the equivalent of the annual global electricity production.

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The Climate Emergency

Insolation (Incoming solar Radiation)

Solar flux at a distance r from the sun:


$$F = \frac{6.33 \times 10^7 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$


$F = 1368 \text{ W/m}^2$ ← solar flux at Earth's orbit

Power intercepted by the Earth: $F \times \pi r_E^2 \text{ W}$
 Earth's surface area: $4\pi r_E^2 \text{ m}^2$

Average surface flux: $\frac{F \times \pi r_E^2}{4\pi r_E^2} = \frac{F}{4} = 342 \text{ W/m}^2$



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The Climate Emergency

Insolation



Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^7 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$


$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$
 $F = 1368 \text{ W/m}^2$

Power intercepted by the Earth:
 $F \times \pi r_E^2 \text{ W}$, r_E = radius of Earth = $6.37 \times 10^6 \text{ m}$
 $F = 1.74 \times 10^{17} \text{ W}$

Biologically Stored Energy
 total coal reserves: 10^{15} kg
 energy content: $3 \times 10^7 \text{ J/kg}$
 total energy in coal reserves: $3 \times 10^{22} \text{ J}$
 = **2 days of insolation**

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The Climate Emergency

Insolation

Global Average Insolation

intercepted flux: $F = 1368 \text{ W/m}^2$
 Earth cross-section: πr_E^2
 surface area: $4\pi r_E^2$
 average flux: $1368/4 = 342 \text{ W/m}^2 = Q$

Simple Model

Assume that Earth is a perfectly thermally conducting black body.


$$Q = \sigma T^4$$

$$T = (Q/\sigma)^{1/4} = (342/5.67 \times 10^{-8})^{1/4}$$


$$= 279\text{K} = 6^\circ\text{C} = 43^\circ\text{F}$$

Dynamics
 $R \frac{dT}{dt} = Q - \sigma T^4$

heat capacity stable equilibrium



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The Climate Emergency

Albedo

Not all the insolation reaches the surface. Some is reflected back into space.
 The proportion reflected is called the albedo, denoted α .
 For Earth, $\alpha \approx 0.3$.


Simple Model

Assume that Earth is a perfectly thermally conducting black body, but only 70% of the insolation is absorbed.

$$T = (0.7 \cdot F/\sigma)^{1/4} = (0.7 \cdot 342/5.67 \times 10^{-8})^{1/4}$$

$$= 255\text{K} = -18^\circ\text{C} = 0^\circ\text{F}$$

Dynamics
 $R \frac{dT}{dt} = Q(1-\alpha) - \sigma T^4$ stable equilibrium



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The Climate Emergency

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Not all the insolation reaches the surface. Some is reflected back into space.
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Simple Model



Assume that Earth is a perfectly thermally conducting black body, but only 70% of the insolation is absorbed.

$$T = (0.7 \cdot F / \sigma)^{1/4} = (0.7 \cdot 342 / 5.67 \times 10^{-8})^{1/4}$$

$$= [255\text{K} = -18^\circ\text{C} = 0^\circ\text{F}]$$

Dynamics
 $R \frac{dT}{dt} = Q(1 - \alpha) - \sigma T^4$ → stable equilibrium

Why isn't the Earth a Snowball?

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The Climate Emergency

Why isn't the Earth a Snowball?

The Greenhouse Effect

Greenhouse gases (CO_2 , H_2O , CH_4) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

Who discovered the greenhouse effect?

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The Climate Emergency

Why isn't the Earth a Snowball?




The Greenhouse Effect

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Who discovered the greenhouse effect?

A mathematician!

Joseph Fourier (1827), *Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires*, *Mémoires de l'Académie Royale des Sciences*, t. vii., p. 569.

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The Climate Emergency

Why isn't the Earth a Snowball?

The Greenhouse Effect!

Joseph Fourier, *Mémoires de l'Académie des Sciences de l'Institut de France*, t. vii. 1827.

Svante Arrhenius, "On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground," *Philosophical Magazine and Journal of Science (Fifth Series)* 41, pp. 237-276, 1896.

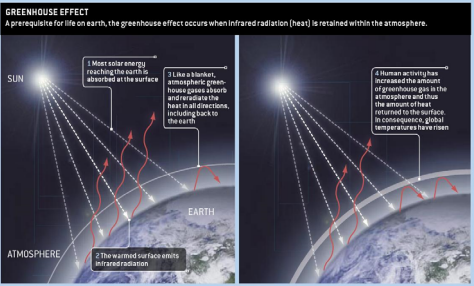





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The Climate Emergency

The Greenhouse Effect

GREENHOUSE EFFECT
A prerequisite for life on earth, the greenhouse effect occurs when infrared radiation (heat) is retained within the atmosphere.



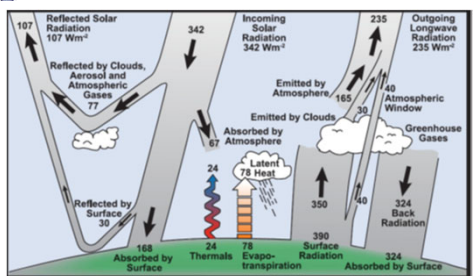
Gary Stix, *Scientific American* September 2006, pp.46-49



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The Climate Emergency

Earth's Heat Balance



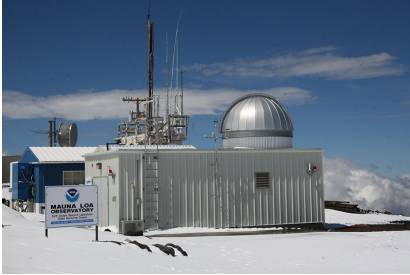
Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

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The Climate Emergency
Can we measure greenhouse gasses?

Mauna Loa Observatory



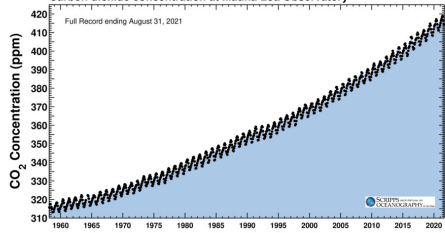
https://research.noaa.gov/Portals/0/EasyDNN/news/1502/200600p587EDNmain10061mlc_sign_miller.jpg

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The Climate Emergency
Can we measure greenhouse gasses?

Keeling Curve

August 28, 2021
Carbon dioxide concentration at Mauna Loa Observatory

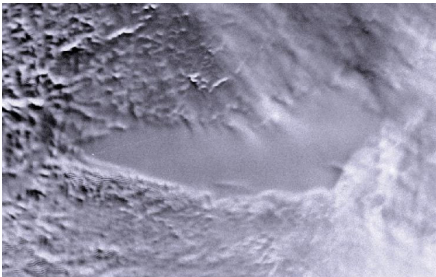


<https://keelingcurve.ucsd.edu/>

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The Climate Emergency
Can we measure greenhouse gasses?

Lake Vostok




<http://svs.gsfc.nasa.gov/vis/a000000/a000900/a000996/index.html>

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The Climate Emergency
Can we measure greenhouse gasses?

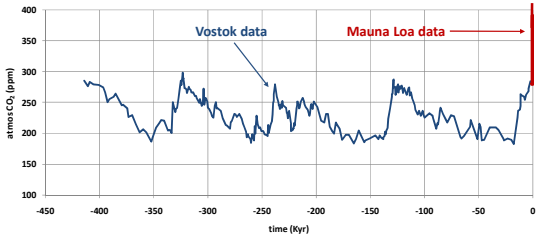
Core Samples
 Photographs copyright Reto Stöckli, NASA GSFC



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The Climate Emergency
Can we measure greenhouse gasses?

Atmospheric CO₂ (Vostok data)




Petit, et al, *Nature* 399 (June 3 1999), pp.429-436

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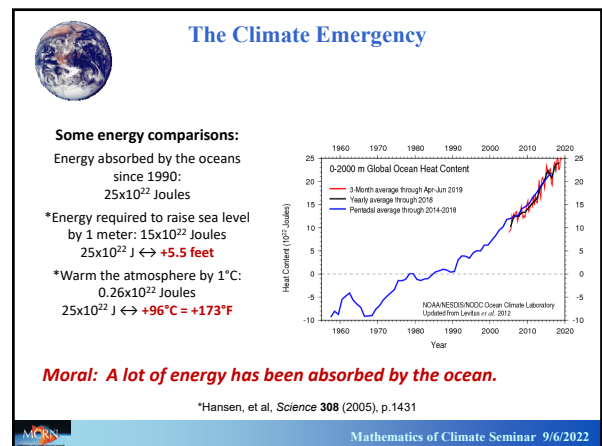
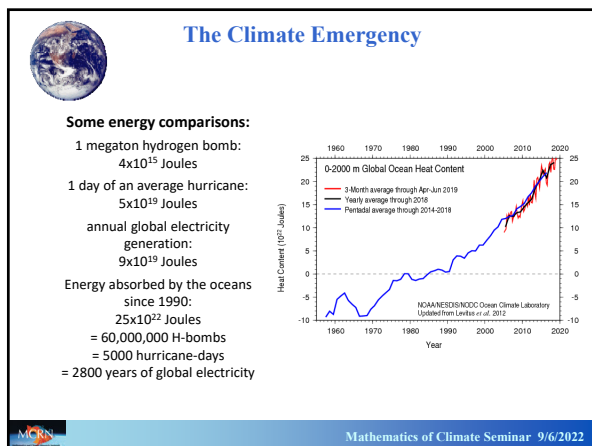
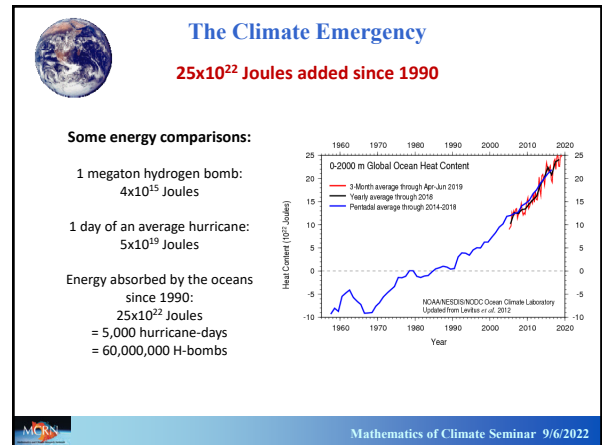
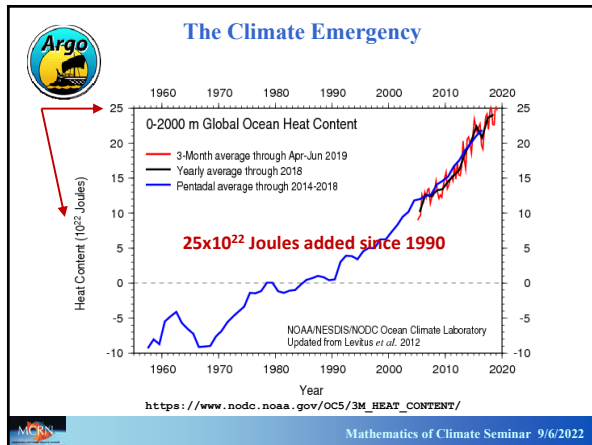
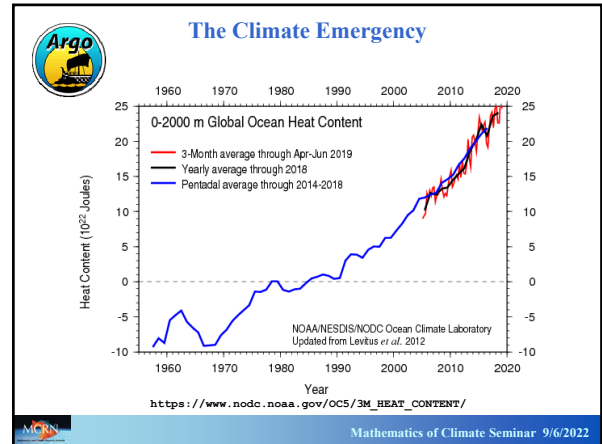
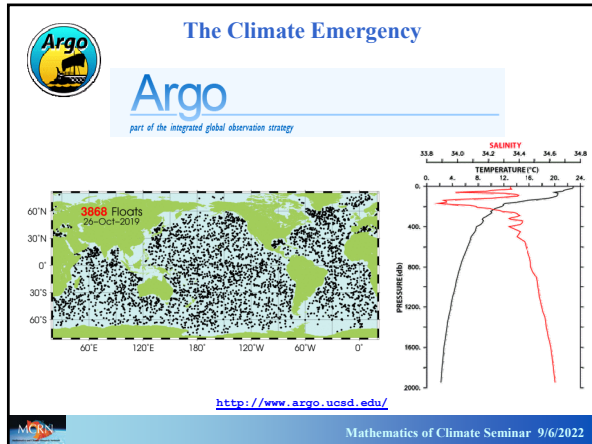
The Climate Emergency
Can we measure heat imbalance?

Argo
 part of the integrated global observation strategy



<http://www.argo.ucsd.edu/>

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The Climate Emergency

Heat Imbalance

Net TOA Radiation (ICERES)
Planetary Heat Uptake (in Situ)

Year

https://www.nasa.gov/sites/default/files/thumbnails/image/figure_1_v2.gif

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The Climate Emergency

Until you start focusing on what needs to be done rather than what is politically possible, there is no hope. We can't solve a crisis without treating it as a crisis. We need to keep the fossil fuels in the ground, and we need to focus on equity. And if solutions within the system are so impossible to find, maybe we should change the system itself.

Greta Thunberg, COP24 Speech, December, 2018

<https://www.youtube.com/watch?v=vFkQSGyeCWg>

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The Climate Emergency

Rey, Star Wars

Emperor Palpatine, Star Wars

I haven't come to lead the Sith. I've come to end them.

Rey, Episode IX – The Rise of Skywalker

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Coming Attraction

An Introduction to Energy Balance Models

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