Earth's Heat Imbalance

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Mathematics of Climate Seminar
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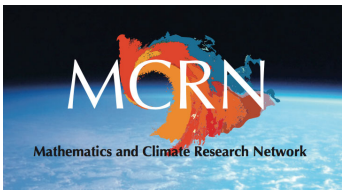

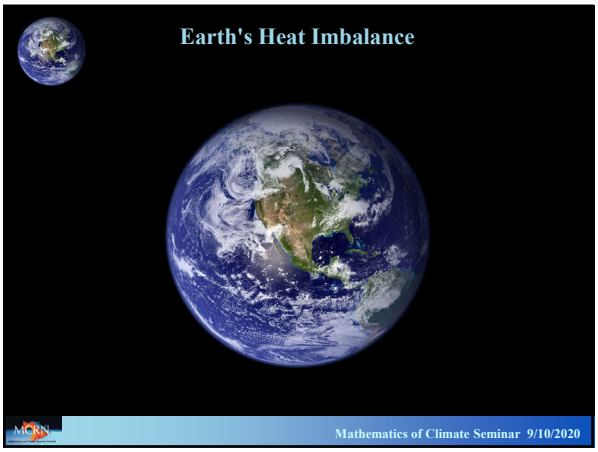

Earth's Heat Imbalance

What Determines Earth's Surface Temperature?

"Energy Balance"

Conservation of Energy

temperature change ~ energy in – energy out

short wave energy from the Sun

long wave energy from the planet

Everything else is detail.

Earth's Heat Imbalance

Stefan-Boltzmann Law

$$F = \sigma T^4$$

power flux (W/m²) — temperature (K)

Stefan-Boltzmann constant

$\sigma \approx 5.67 \times 10^{-8}\ \mathrm{W/m^2 K^4}$

Reasonable approximation:
Every body in the solar system radiates energy according to this law.

Earth's Heat Imbalance

Stefan-Boltzmann Law

$$F = \sigma T^4$$

power flux (W/m²) — temperature (K)

Stefan-Boltzmann constant

$\sigma \approx 5.67 \times 10^{-8}\ \mathrm{W/m^2 K^4}$

Example

surface temperature of the Sun: 5780K

power flux: $5.67 \times 10^{-8} \times (5780)^4 = 6.33 \times 10^7\ \mathrm{W/m^2}$

total solar power output: $6.33 \times 10^7 \times 4\pi (r_S)^2$,
where r_S = radius of the sun = 6.96×10^8 m

total solar output: 3.85×10^{26} W

230 nanoseconds = time it takes for the Sun to produce the equivalent of the annual global electricity production.

Earth's Heat Imbalance

Insolation
(**In**coming **sol**ar Radi**ation**)

Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^7\, 4\pi r_S^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_S}{r}\right)^2\ \mathrm{W/m^2}$$

$r_S = 6.96 \times 10^8$ m
$r = 1.5 \times 10^{11}$ m

$\boxed{F = 1368\ \mathrm{W/m^2}}$ ← solar flux at Earth's orbit

Power intercepted by the Earth: $F \times \pi r_E^2$ W

Earth's surface area: $4\pi r_E^2$ m²

Average surface flux: $\dfrac{F \times \pi r_E^2}{4\pi r_E^2} = \dfrac{F}{4} = \boxed{342\ \mathrm{W/m^2}}$

Earth's Heat Imbalance

Insolation

Solar flux at a distance r from the sun:

$$F = \frac{6.33\times10^7\, 4\pi r_S^2}{4\pi r^2} = 6.33\times10^7 \left(\frac{r_S}{r}\right)^2 \text{ W/m}^2$$

$r_S = 6.96\text{x}10^8$ m
$r = 1.5\text{x}10^{11}$ m
$F = 1368$ W/m^2

Power intercepted by the Earth:

$F \times \pi r_E^2$ W, $r_E =$ radius of Earth $= 6.37\times10^6$ m

$F = 1.74\times10^{17}$ W

Biologically Stored Energy

total coal reserves: 10^{15} kg
energy content: $3\text{x}10^7$ J/kg
total energy in coal reserves: $3\text{x}10^{22}$ J

= ***2 days of insolation***

Earth's Heat Imbalance

Insolation

Global Average Insolation
intercepted flux: $F = 1368$ W/m^2
Earth cross-section: πr_E^2
surface area: $4\pi r_E^2$
average flux: $1368/4 = 342$ W/m$^2 = Q$

Simple Model

Assume that Earth is a perfectly thermally conducting black body.

$$Q = \sigma T^4$$

$$T = (Q/\sigma)^{1/4} = \left(342/5.67\times10^{-8}\right)^{1/4}$$

$$= \boxed{279\text{K} = 6^\circ\text{C} = 43^\circ\text{F}}$$

Dynamics

$$R\frac{dT}{dt} = Q - \sigma T^4$$

heat capacity

stable equilibrium

Earth's Heat Imbalance

Albedo

Not all the insolation reaches the surface. Some is reflected back into space.
The proportion reflected is called the albedo, denoted α.

For Earth, $\alpha \approx 0.3$.

Simple Model

Assume that Earth is a perfectly thermally conducting black body, but only 70% of the insolation is absorbed.

$$T = (0.7\cdot F/\sigma)^{1/4} = \left(0.7\cdot 342/5.67\times10^{-8}\right)^{1/4}$$

$$= \boxed{255\text{K} = -18^\circ\text{C} = 0^\circ\text{F}}$$

Dynamics

$$R\frac{dT}{dt} = Q(1-\alpha) - \sigma T^4$$

stable equilibrium

Earth's Heat Imbalance

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Dynamics

$$R\frac{dT}{dt} = Q(1-\alpha) - \sigma T^4$$

stable equilibrium

Why isn't the Earth a Snowball?

Earth's Heat Imbalance

The Greenhouse Effect

Greenhouse gases (CO_2, H_2O, CH_4) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

Who discovered the greenhouse effect?

Earth's Heat Imbalance

The Greenhouse Effect

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Who discovered the greenhouse effect?

Joseph Fourier (1827), Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires, *Mémoires de l'Académie Royale des Sciences*, t. vii., p. 569.

Earth's Heat Imbalance

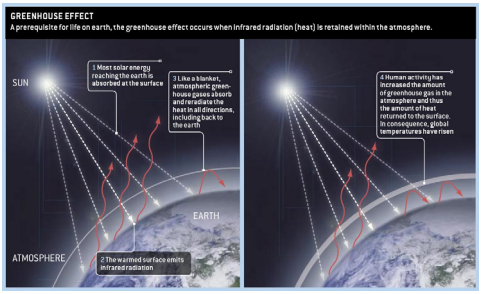
Why isn't the Earth a Snowball?

The Greenhouse Effect!

Joseph Fourier, *Mémoires de l'Académie des Sciences de l'Institut de France*, t. vii. 1827.

Svante Arrhenius, "On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground," *Philosophical Magazine and Journal of Science (Fifth Series)* **41**, pp. 237-276, 1896.

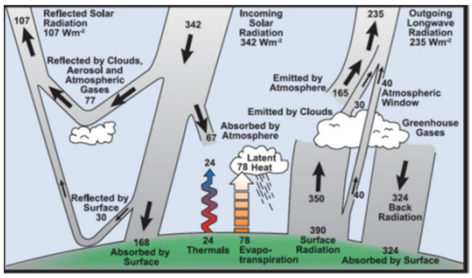
Earth's Heat Imbalance



Gary Stix, *Scientific American* September 2006, pp.46-49

Earth's Heat Imbalance

Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

Earth's Heat Imbalance

OLR as a Function of Surface Temperature
(Outgoing Longwave Radiation)

$$\text{OLR} \approx A + BT$$

A and B are determined from satellite observations.
T is surface temperature (*in Celsius*).

$$A = 202 \text{ W/m}^2$$
$$B = 1.90 \text{ W/m}^2\text{K}$$

Kelvin → Dynamics

$$R\frac{dT}{dt} = Q(1-\alpha) - \sigma T^4$$

T: photosphere temperature

becomes

Celsius →

$$R\frac{dT}{dt} = Q(1-\alpha) - (A + BT)$$

T: global mean surface temperature

Earth's Heat Imbalance

OLR as a Function of Surface Temperature

$$\text{OLR} \approx A + BT$$

Important:
A+BT is **not** a linear approximation to the Stefan-Boltzmann equation.

different

Kelvin → Dynamics

$$R\frac{dT}{dt} = Q(1-\alpha) - \sigma T^4$$

photosphere temperature

Celsius → becomes

$$R\frac{dT}{dt} = Q(1-\alpha) - (A + BT)$$

global mean surface temperature

Earth's Heat Imbalance

Homogeneous Earth

$$R\frac{dT}{dt} = Q(1-\alpha) - (A + BT)$$

Equilibrium Temperature: $Q(1-\alpha) - A - BT_{eq} = 0$

$$T_{eq} = \frac{Q(1-\alpha) - A}{B}$$ Stable, since $B > 0$.

Ice-free planet: $\alpha = 0.32$, $T_{eq} = 16$ °C
Snowball planet: $\alpha = 0.62$, $T_{eq} = -38$ °C

No glacier would form on an ice-free Earth.

No glacier would melt on a snowball Earth.

Easy question:
Why do we have ice caps?

Hard question:
If Earth was ever a snowball, how did we get out?

Earth's Heat Imbalance

Latitude Dependence

Make T depend on y = sin(latitude)

$$R\frac{\partial T(y,t)}{\partial t} = Qs(y)(1-\alpha) - (A + BT(y,t))$$

insolation distribution

Q = global annual average insolation = 342 W/m^2

$s(y)$ = distribution across latitudes $\left(\int_0^1 s(y)dy = 1\right)$

One can show that

$$s(y) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\theta - y\cos\beta\right)^2}\,d\theta$$

β = obliquity = 23.4°

Chylek and Coakley's quadratic approximation:

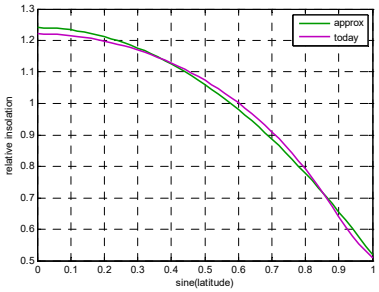
$$s(y) \approx 1 - 0.241\left(3y^2 - 1\right)$$

Earth's Heat Imbalance

Insolation Distribution

green = quadratic approximation (Chylek & Coakley)

fuchsia = formula using obliquity of 23.4°



Earth's Heat Imbalance

Latitude Dependence

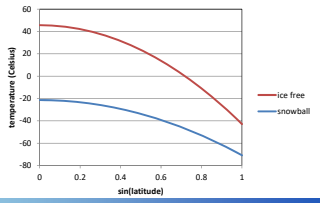
$$R\frac{\partial T(y,t)}{\partial t} = Qs(y)(1-\alpha) - (A + BT(y,t))$$

Note that y is just a parameter.

Equilibrium Temperature Profile

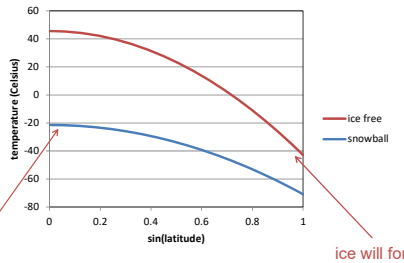
$$T_{eq}(y) = \frac{Qs(y)(1-\alpha) - A}{B}$$

α = 0.32: ice free
α = 0.62: snowball



Earth's Heat Imbalance

Latitude Dependence



ice won't melt (no exit from snowball)

ice will form (icecap)

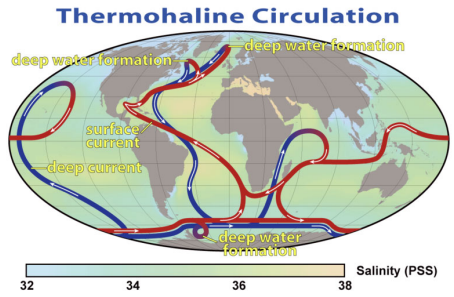
Earth's Heat Imbalance

What's Missing?

$$R\frac{\partial T(y,t)}{\partial t} = Qs(y)(1-\alpha) - (A + BT(y,t))$$

Earth's Heat Imbalance

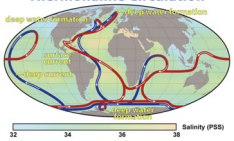
What's Missing?



Earth's Heat Imbalance

What's Missing?

Thermohaline Circulation



Earth's Heat Imbalance

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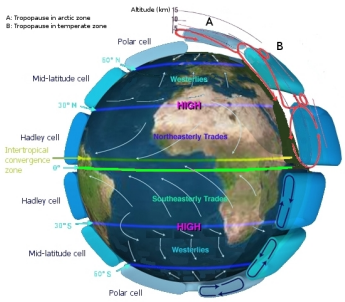

Earth's Heat Imbalance

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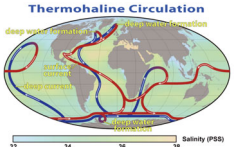
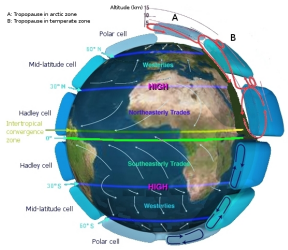




Earth's Heat Imbalance

What's Missing?

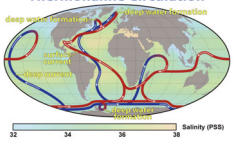
https://www.nytimes.com/2017/09/04/us/hurricane-irma.html

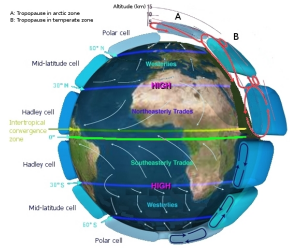
Earth's Heat Imbalance

What's Missing?

Weather!

Thermohaline Circulation





Earth's Heat Imbalance

Budyko's Model

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + \boxed{C(\overline{T}-T)}$$

Weather

global mean temperature $\overline{T}(t) = \int_0^1 T(y,t)dy$

Second Law of Thermodynamics:
Energy travels from hot places to cold places.

Mikhail Budyko

Budyko's equation as a dynamical system:
T lives in a function space (temperature as a function of latitude).

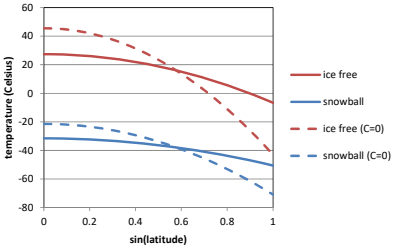
Earth's Heat Imbalance

Budyko's Equilibrium

$$Qs(y)(1-\alpha(y)) - (A + BT^*(y)) + C(\overline{T}^* - T^*(y)) = 0$$

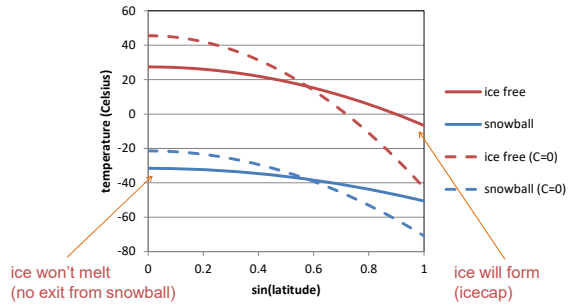
Equilibrium temperature profile: $T^*(y) = \frac{1}{B+C}(Qs(y)(1-\alpha(y)) - A + C\overline{T}^*)$

C = 3.04
$\alpha(y)$ = 0.32: ice free
$\alpha(y)$ = 0.62: snowball
(constant albedo)



Earth's Heat Imbalance

Budyko's Equilibrium



Earth's Heat Imbalance

Budyko's Model

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A + BT) + C(\overline{T} - T)$$

So far, we have looked only at equilibria.
What about the dynamics?

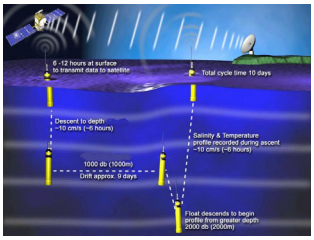
temperature change ~ energy in – energy out

Energy Imbalance

Is there an imbalance?

Earth's Heat Imbalance

Argo
part of the integrated global observation strategy



http://www.argo.ucsd.edu/

Earth's Heat Imbalance

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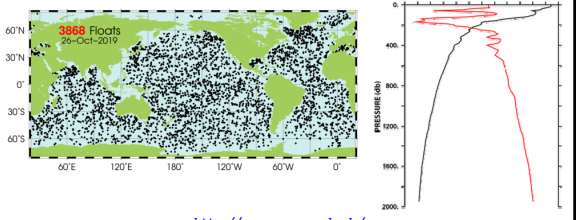

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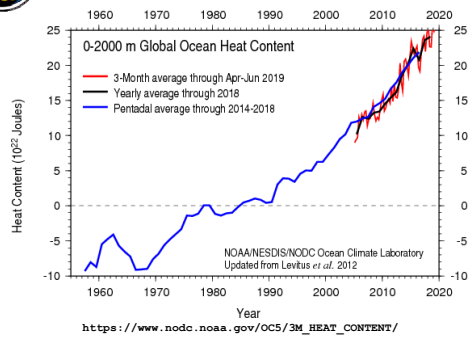
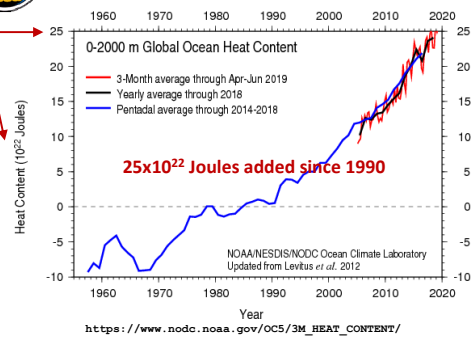
Earth's Heat Imbalance



https://www.nodc.noaa.gov/OC5/3M_HEAT_CONTENT/

Earth's Heat Imbalance



https://www.nodc.noaa.gov/OC5/3M_HEAT_CONTENT/

Earth's Heat Imbalance

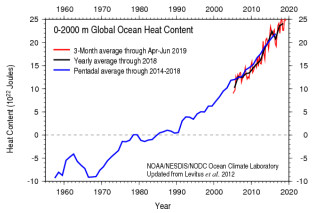
25x10²² Joules added since 1990

Some energy comparisons:

1 megaton hydrogen bomb:
$4x10^{15}$ Joules

1 day of an average hurricane:
$5x10^{19}$ Joules

Energy absorbed by the oceans since 1990:
$25x10^{22}$ Joules
= 5,000 hurricane-days
= 60,000,000 H-bombs



Earth's Heat Imbalance

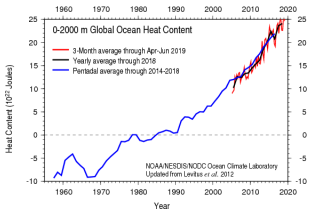
Some energy comparisons:

1 megaton hydrogen bomb:
$4x10^{15}$ Joules

1 day of an average hurricane:
$5x10^{19}$ Joules

annual global electricity generation:
$9x10^{19}$ Joules

Energy absorbed by the oceans since 1990:
$25x10^{22}$ Joules
= 60,000,000 H-bombs
= 5000 hurricane-days
= 2800 years of global electricity



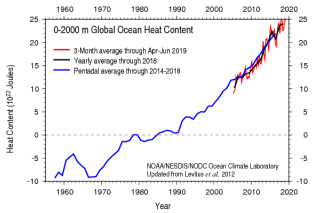
Earth's Heat Imbalance

Some energy comparisons:

Energy absorbed by the oceans since 1990:
$25x10^{22}$ Joules

*Energy required to raise sea level by 1 meter: $15x10^{22}$ Joules
$25x10^{22}$ J ↔ **+5.5 feet**

*Warm the atmosphere by 1°C:
$0.26x10^{22}$ Joules
$25x10^{22}$ J ↔ **+96°C = +173°F**



Moral: A lot of energy has been absorbed by the ocean.

*Hansen, et al, *Science* **308** (2005), p.1431

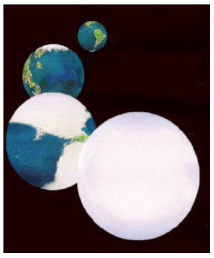
Earth's Heat Imbalance

What can we do with Budyko's equation?

Earth's Heat Imbalance

What can we do with Budyko's equation?

Speculate about Snowball Earth.



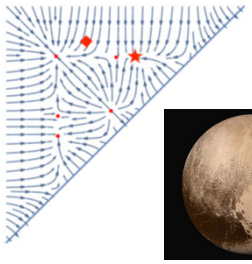
Esther Widiasih
PhD 2010

Anna Barry
Postdoc 2012-14

Earth's Heat Imbalance

What can we do with Budyko's equation?

Explain Pluto's heart.

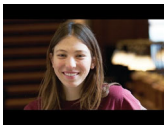


Alice Nadeau, PhD 2019

Earth's Heat Imbalance

What can we do with Budyko's equation?

Predict permafrost melt.



Aileen Zebrowski, BS 2016

Kaitlin Hill, Postdoc 2017-19

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