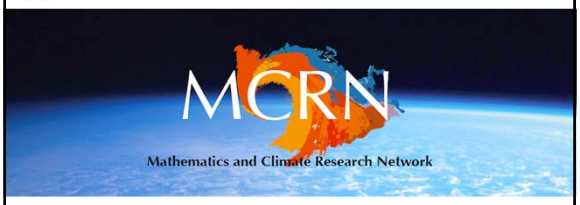
Math and Climate Seminar IMA




**Joint MCRN/IMA Math and Climate Seminar**
**Tuesdays 11:15 – 12:05**
**streaming video available at**
**www.ima.umn.edu**

MCRN
www.mathclimate.org

# Budyko's Model Forced by Milankovitch Cycles

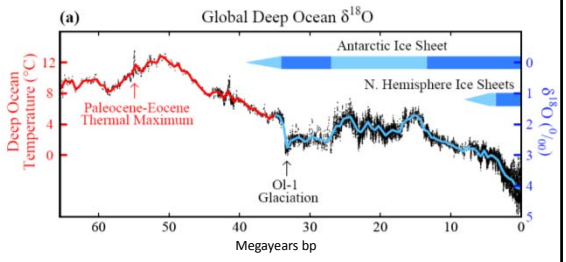
Richard McGehee

Seminar on the Mathematics of Climate
IMA, MCRN, School of Mathematics
January 22, 2013

## Milankovitch and Budyko

**Paleoclimate**

**Temperatures in the Cenozoic Era**




Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* **2** (2008)

## Milankovitch and Budyko

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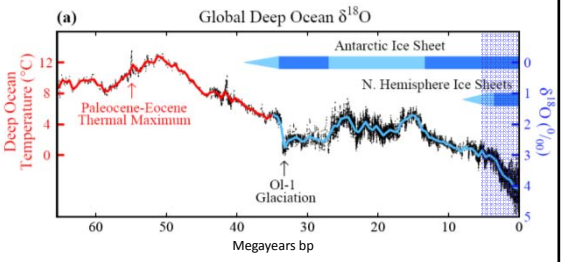


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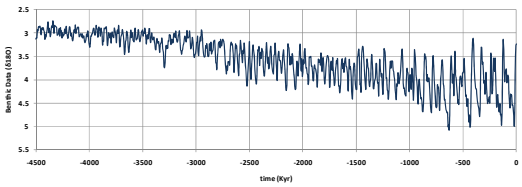
## Milankovitch and Budyko

**Paleoclimate**

**$^{18}O$ in Foraminifera Fossils During the Past 4.5 Myr**




Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

## Milankovitch and Budyko

**Paleoclimate**

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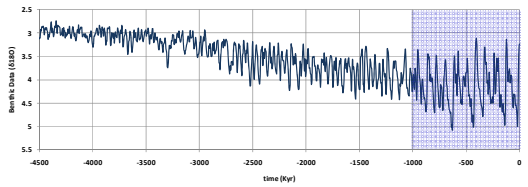
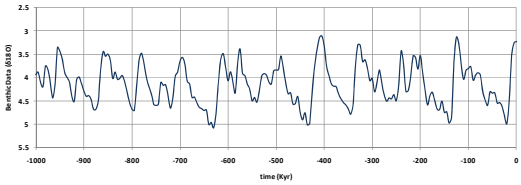



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

## Milankovitch and Budyko

**Paleoclimate**

**$^{18}O$ in Foraminifera Fossils During the Past 1.0 Myr**



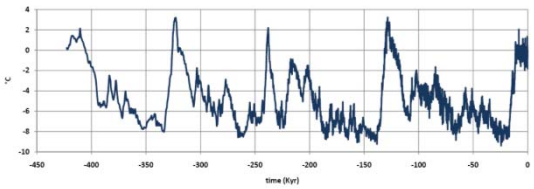

Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

## Milankovitch and Budyko

**Paleoclimate**

**Recent (last 400 Kyr) Temperature Cycles**

**Vostok Ice Core Data**




J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

## Milankovitch and Budyko

**Paleoclimate**

**What Causes Glacial Cycles?**

**Widely Accepted Hypothesis**

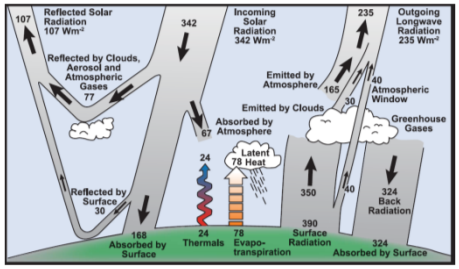
The glacial cycles are driven by the variations in the Earth's orbit (***Milankovitch Cycles***), causing a variation in incoming solar radiation (***insolation***).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.

## Milankovitch and Budyko

**Heat Balance**

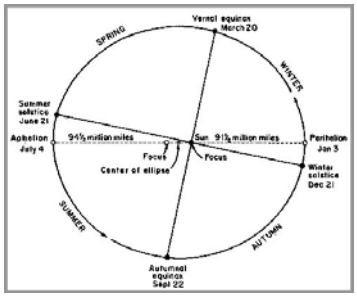



*Historical Overview of Climate Change Science*, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

## Milankovitch and Budyko

**Milankovitch Cycles**

**Eccentricity**

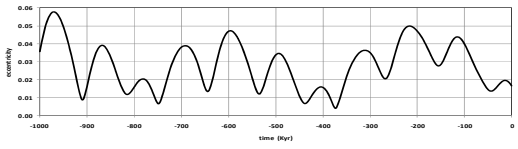



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery,* Harvard Univ. Press, 1979.

## Milankovitch and Budyko

**Milankovitch Cycles**

**Eccentricity**




Note periods of about 100 Kyr and 400 Kyr.

The effect due to eccentricity is more significant, but not that much:

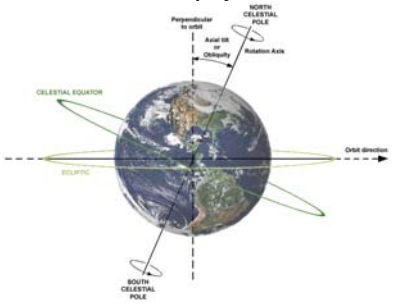
As $e$ varies between 0 and 0.06, $(1-e^2)^{-1/2}$ varies between 1 and 1.0018, or about 0.2%. (Twenty times the effect due to $a$.)

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

## Milankovitch and Budyko

**Milankovitch Cycles**

### Obliquity

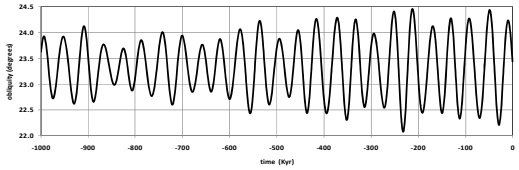



http://upload.wikimedia.org/wikipedia/commons/6/61/AxialTiltObliquity.png

## Milankovitch and Budyko

**Milankovitch Cycles**

### Obliquity



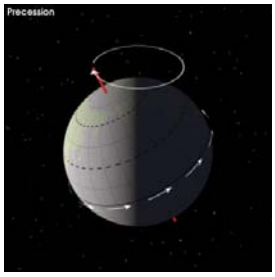

Note period of about 41 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

## Milankovitch and Budyko

**Milankovitch Cycles**

### Precession

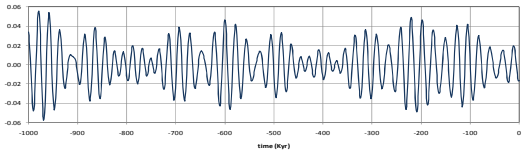



http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html

## Milankovitch and Budyko

**Milankovitch Cycles**

### Precession Index




index = $e \sin\rho$, where $e$ = eccentricity and $\rho$ = precession angle (measured from spring equinox)

Note period of about 23 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

## Milankovitch and Budyko

**Milankovitch Cycles**

### Who was Milankovitch?

*Milutin Milankovitch* was a Serbian mathematician and professor at the University of Belgrade.

In 1920 he published his seminal work on insolation, Earth's orbital parameters, and glacial cycles.

In 1941 he published a book explaining his entire theory.

His work was not fully accepted until 1976.


## Milankovitch and Budyko

**Milankovitch Cycles**

### What happened in 1976?

Hays, Imbrie, and Shackleton, "Variations in the Earth's Orbit: Pacemaker of the Ice Ages," *Science* **194**, 10 December 1976.

James D. Hays

John Imbrie

Nicholas Shackleton

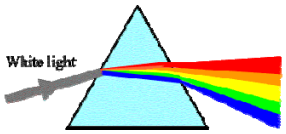
**"It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages."**

## Milankovitch and Budyko

**Milankovitch Cycles**

**Fourier Transform (Power Spectrum)**

**Refraction through a prism**




time series

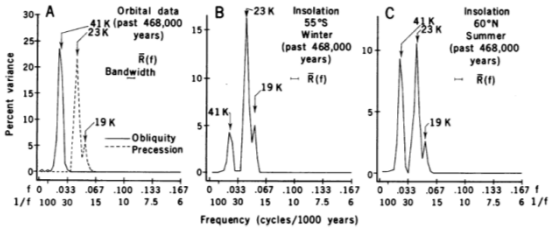
power spectrum

Fourier Transform

## Milankovitch and Budyko

**Milankovitch Cycles**

**Solar Forcing (Hays, et al)**

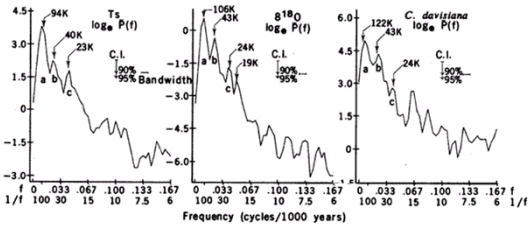



Hays, *et al*, *Science* **194** (1976), p. 1125

## Milankovitch and Budyko

**Milankovitch Cycles**

**Climate Response, Hays, et al**



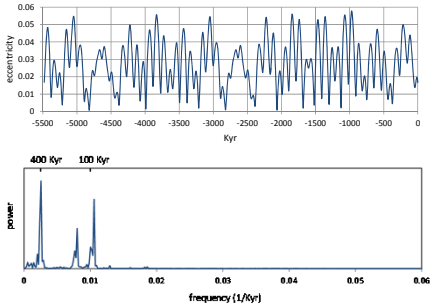

Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1125

## Milankovitch and Budyko

**Milankovitch Cycles**

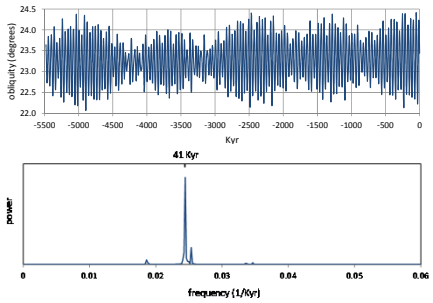
**Eccentricity**




## Milankovitch and Budyko

**Milankovitch Cycles**

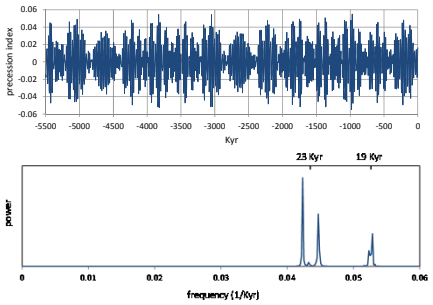
**Obliquity**




## Milankovitch and Budyko

**Milankovitch Cycles**

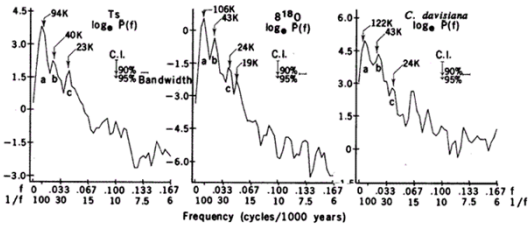
**Precession Index**



## Milankovitch and Budyko

**Milankovitch Cycles**

**Climate Response, Hays, et al**



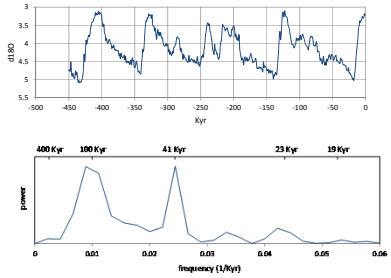

Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1125

## Milankovitch and Budyko

**Milankovitch Cycles**

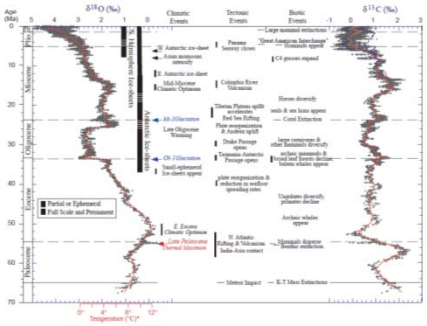
**Hays with Modern Data**




## Milankovitch and Budyko

**Milankovitch Cycles**

Cenozoic Era

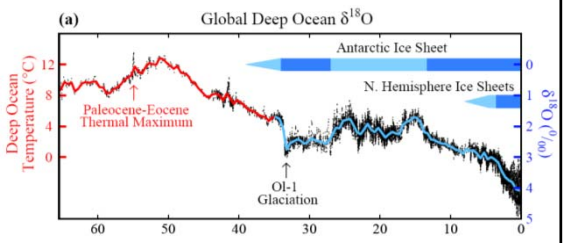


Zachos, *et al*, *Science* **292** (2001), p. 689

## Milankovitch and Budyko

**Milankovitch Cycles**

**Temperatures in the Cenozoic Era**



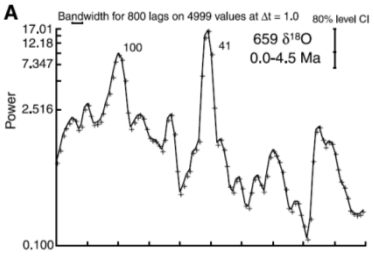

Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

## Milankovitch and Budyko

**Milankovitch Cycles**

**Climate Response (Zachos, et al)**

Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.

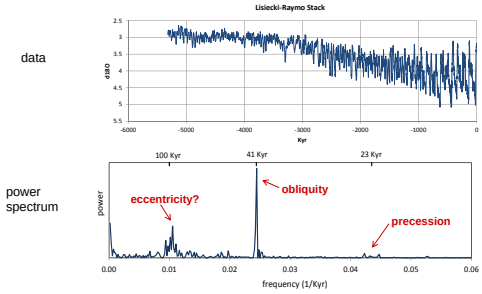



Zachos, *et al*, *Science* **292** (2001), p. 689

## Milankovitch and Budyko

**Milankovitch Cycles**

**$^{18}O$ in Foraminifera Fossils During the Past 5.3 Myr**




Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,**20**, PA1003, doi:10.1029/2004PA001071.

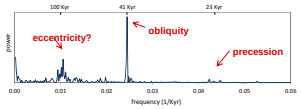
## Milankovitch and Budyko

### Milankovitch Cycles

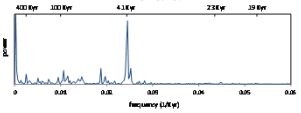
**Last Million Years is Different**

A transition occurred about one million years ago: the amplitude increased and the dominant period changed from 41 kyr to 100 kyr.

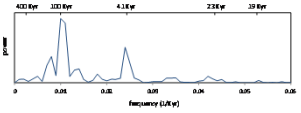
-5 to 0 Myr




-5 to -1 Myr




-1 to 0 Myr




## Milankovitch and Budyko

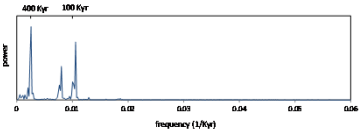
### Milankovitch Cycles

**What's up with the Last Million Years?**

***100,000 Year Problem:*** Why did the eccentricity signal become so dominant during the last million years?

***400,000 Year Problem:*** If the last million years is dominated by eccentricity, what happened to the 400,000 year cycle?

eccentricity

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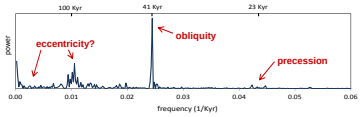
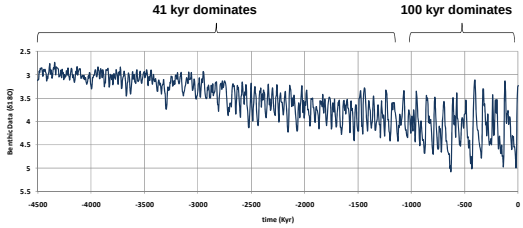



## Milankovitch and Budyko

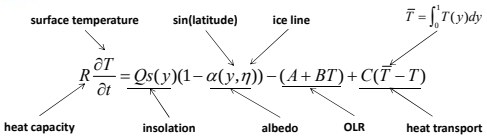
### Milankovitch Cycles




What's up with the last million years?
Did eccentricity reassert itself? Or something else?

## Milankovitch and Budyko

### Budyko's Model




$$\bar{T} = \int_0^1 T(y)dy$$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\bar{T}-T)$$

reduces to

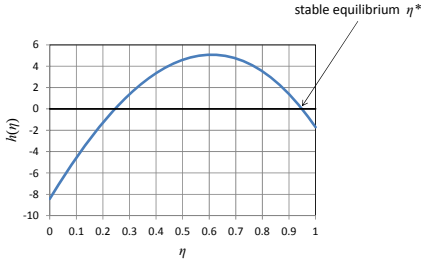
$$\frac{d\eta}{dt} = \varepsilon\left(T(\eta) - T_c\right) \equiv h(\eta)$$

See last semester's handouts.

## Milankovitch and Budyko

### Budyko's Model

$$\frac{d\eta}{dt} = h(\eta)$$




## Milankovitch and Budyko

### Budyko's Model

$$\frac{d\eta}{dt} = h(\eta)$$

The function $h$, and hence the equilibrium solution $\eta^*$, depends on all the parameters of the Budyko model.

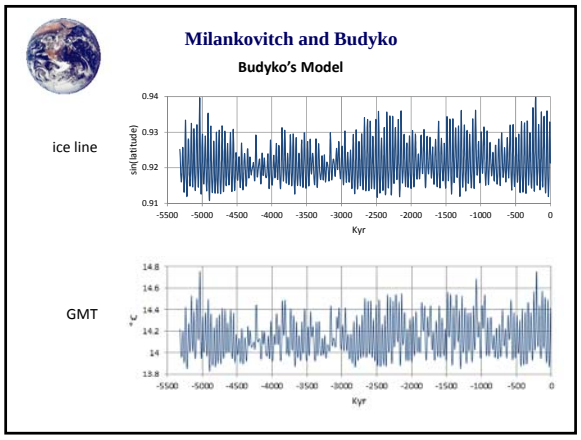
$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\bar{T}-T)$$

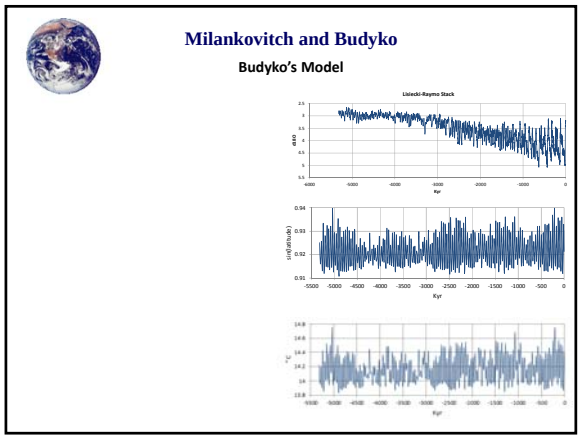
In particular, $\eta^*$ depends on $Q$ and $s(y)$, which depend on the eccentricity $e$ and the obliquity $\beta$.

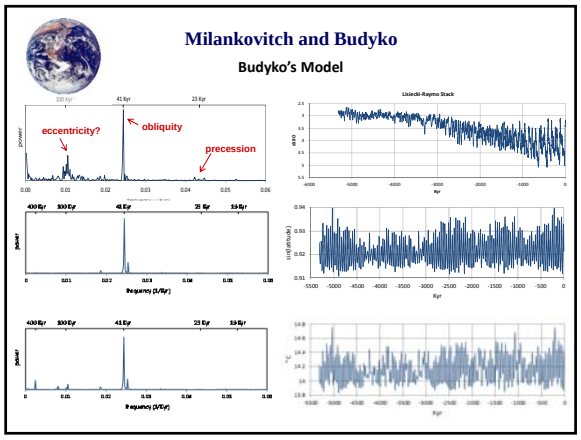
$$Q(e) = \frac{Q_0}{\sqrt{1-e^2}}$$

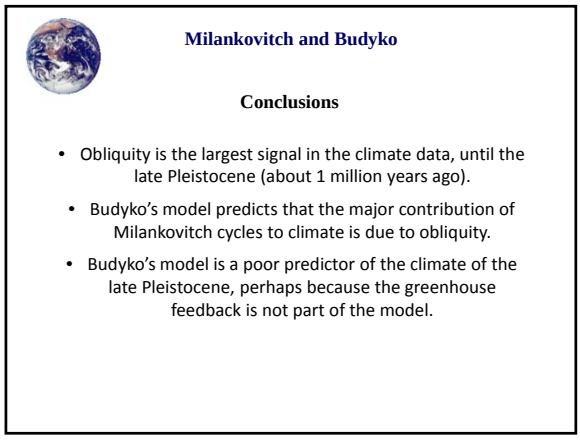
$$s(y,\beta) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\theta - y\cos\beta\right)^2}\,d\theta$$

$$\eta^* = \eta^*(e,\beta)$$


Milankovitch and Budyko
Budyko's Model
ice line
GMT
sin(latitude)
Kyr
°C


Milankovitch and Budyko
Budyko's Model
Lisiecki-Raymo Stack
sin(latitude)
Kyr


Milankovitch and Budyko
Budyko's Model
eccentricity?
obliquity
precession
Lisiecki-Raymo Stack
power
Frequency (1/Kyr)
sin(latitude)
Kyr


Milankovitch and Budyko
Conclusions
• Obliquity is the largest signal in the climate data, until the late Pleistocene (about 1 million years ago).
• Budyko's model predicts that the major contribution of Milankovitch cycles to climate is due to obliquity.
• Budyko's model is a poor predictor of the climate of the late Pleistocene, perhaps because the greenhouse feedback is not part of the model.