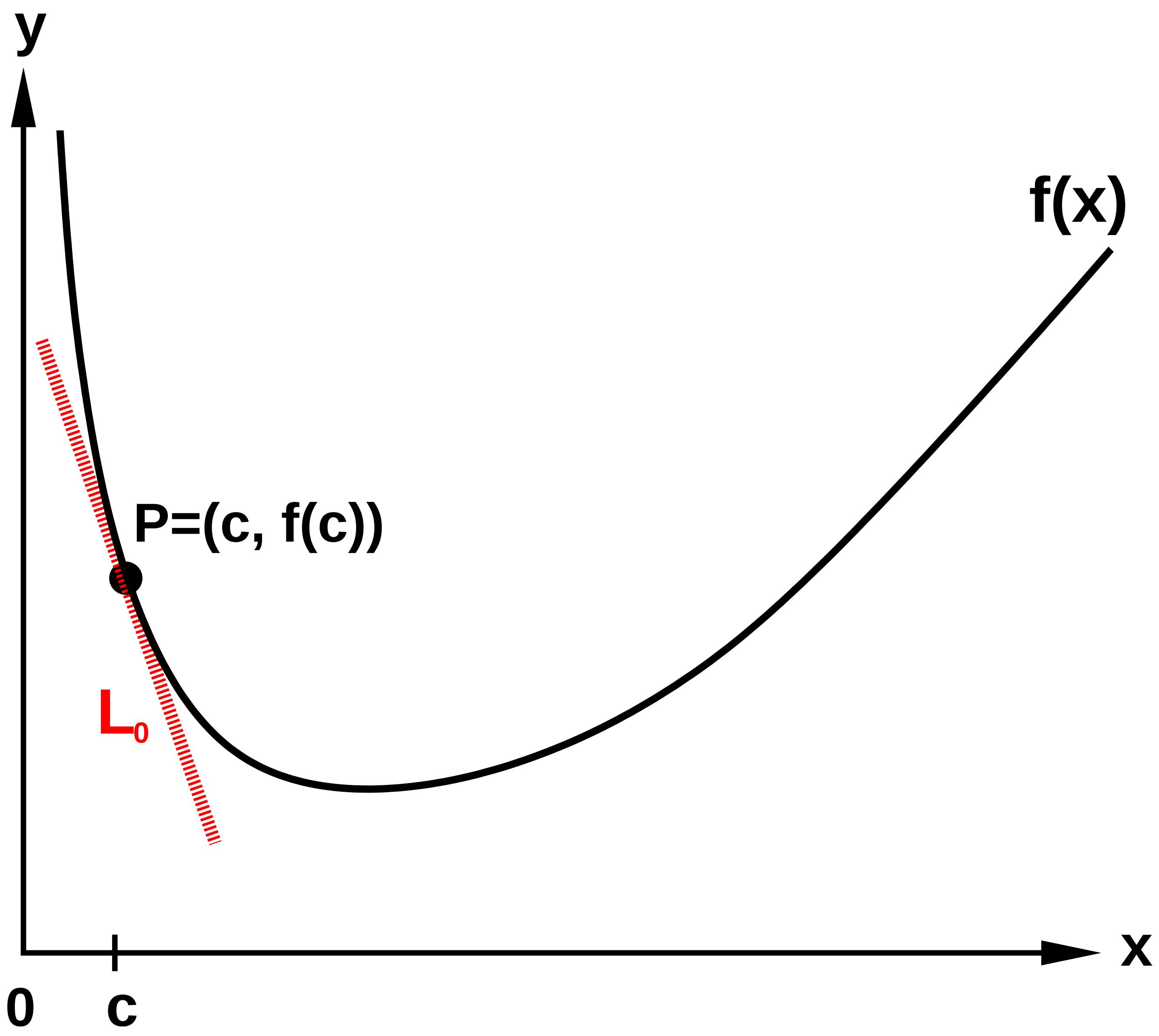
Differentiation

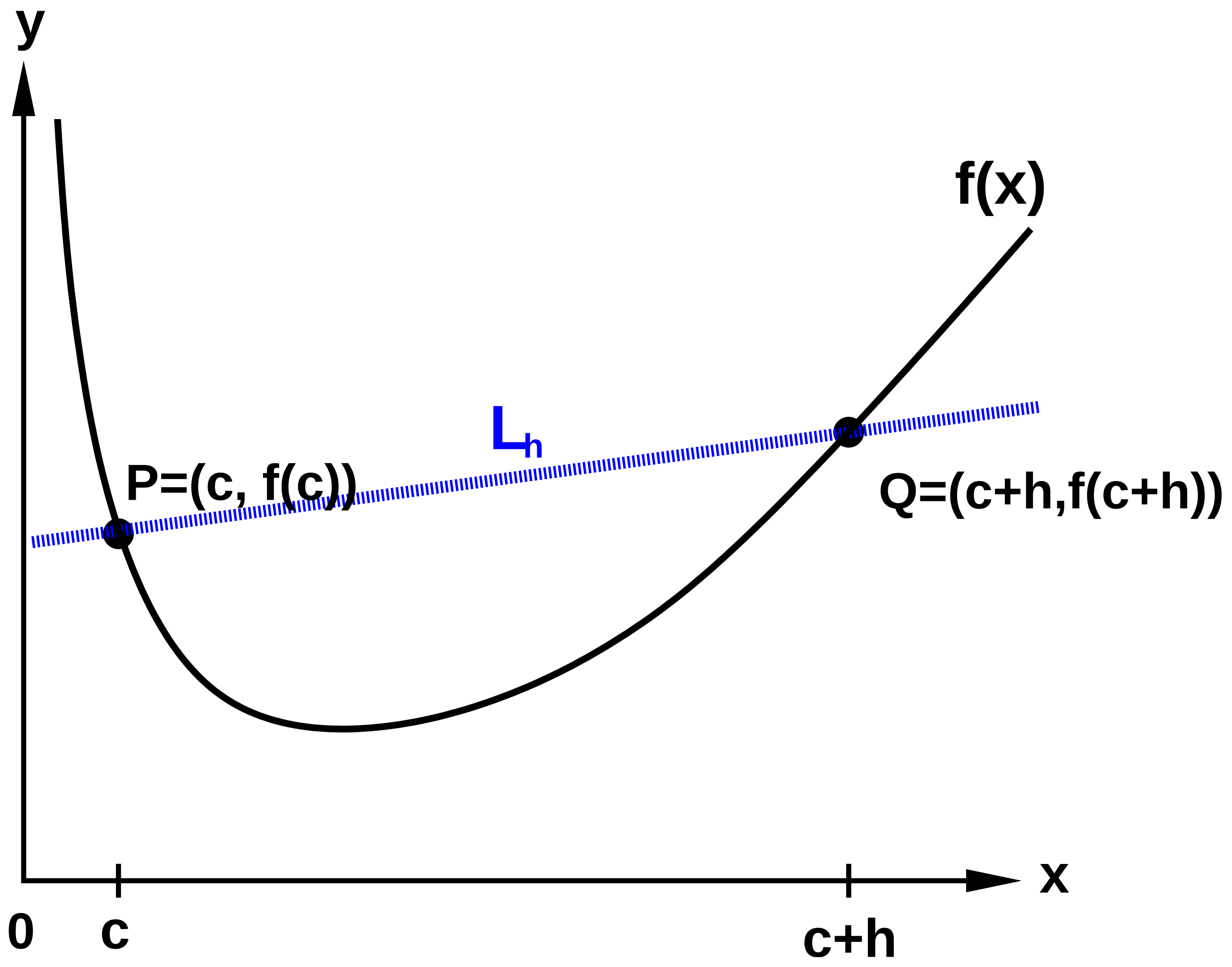
Main Idea: Finding the rate of change of one quantity with respect to another at a particular instant.

Tangent line



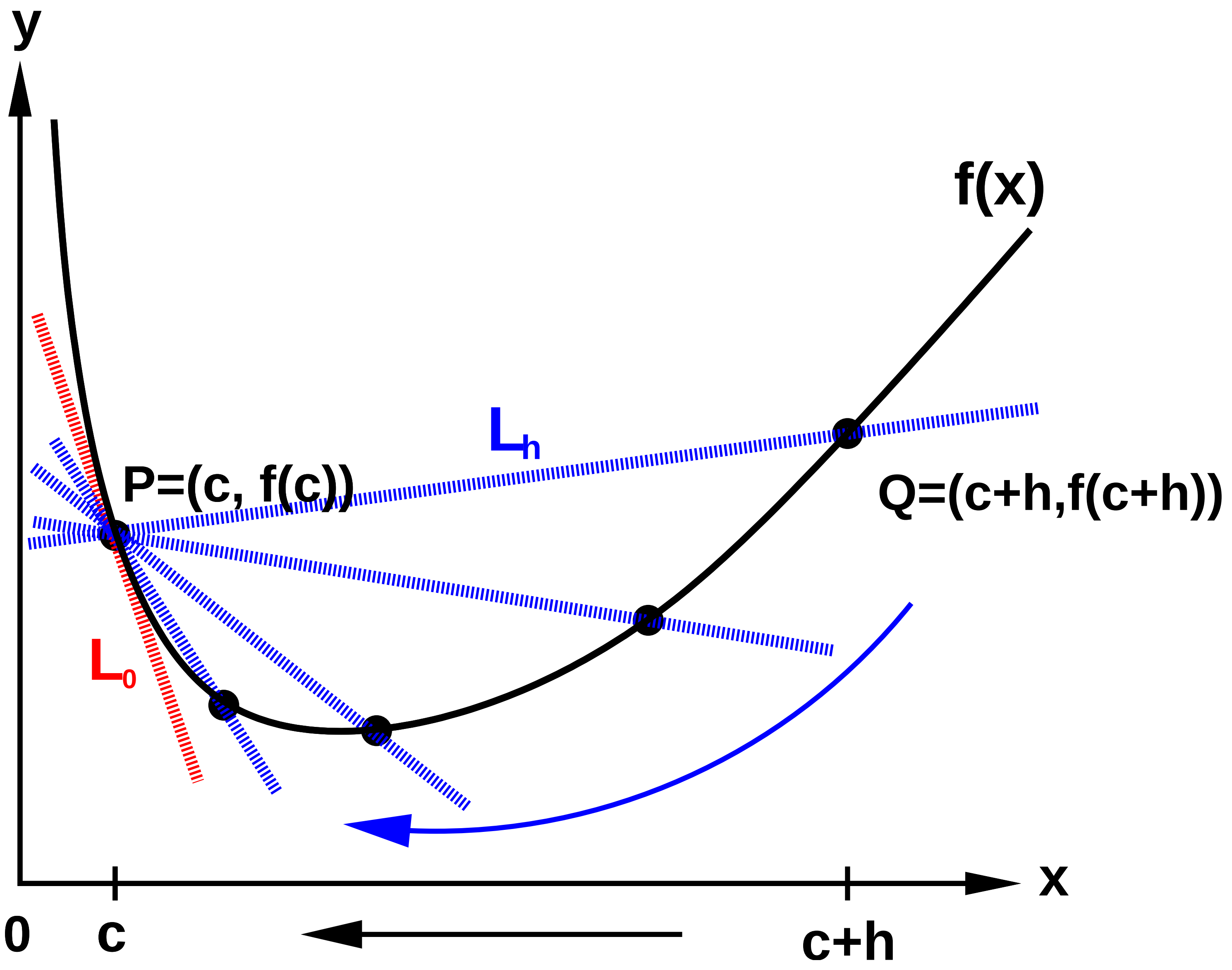
L_0 is the tangent line at the point **P**.

Secant line



L_h is the secant line through the points **P** and **Q**.

The secant lines approach the tangent line.



As $\mathbf{h} \longrightarrow \mathbf{0}$, the secant lines $\mathbf{L_h}$ approach the tangent line $\mathbf{L_0}$.

Slopes of the secant lines and the tangent line

m_{sec} = the slope of the secant line $\mathbf{L_h}$.

m_{tan} = the slope of the tangent line $\mathbf{L_0}$.

As $\mathbf{h} \longrightarrow \mathbf{0}$, $m_{sec} \longrightarrow m_{tan}$ (since $\mathbf{L_h} \longrightarrow \mathbf{L_0}$).

In other words,

$$m_{tan} = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}.$$

The definition of a derivative

The derivative of a function f(x) w.r.t. x is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

A function is differentiable at x=c if the following limit exists

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}.$$

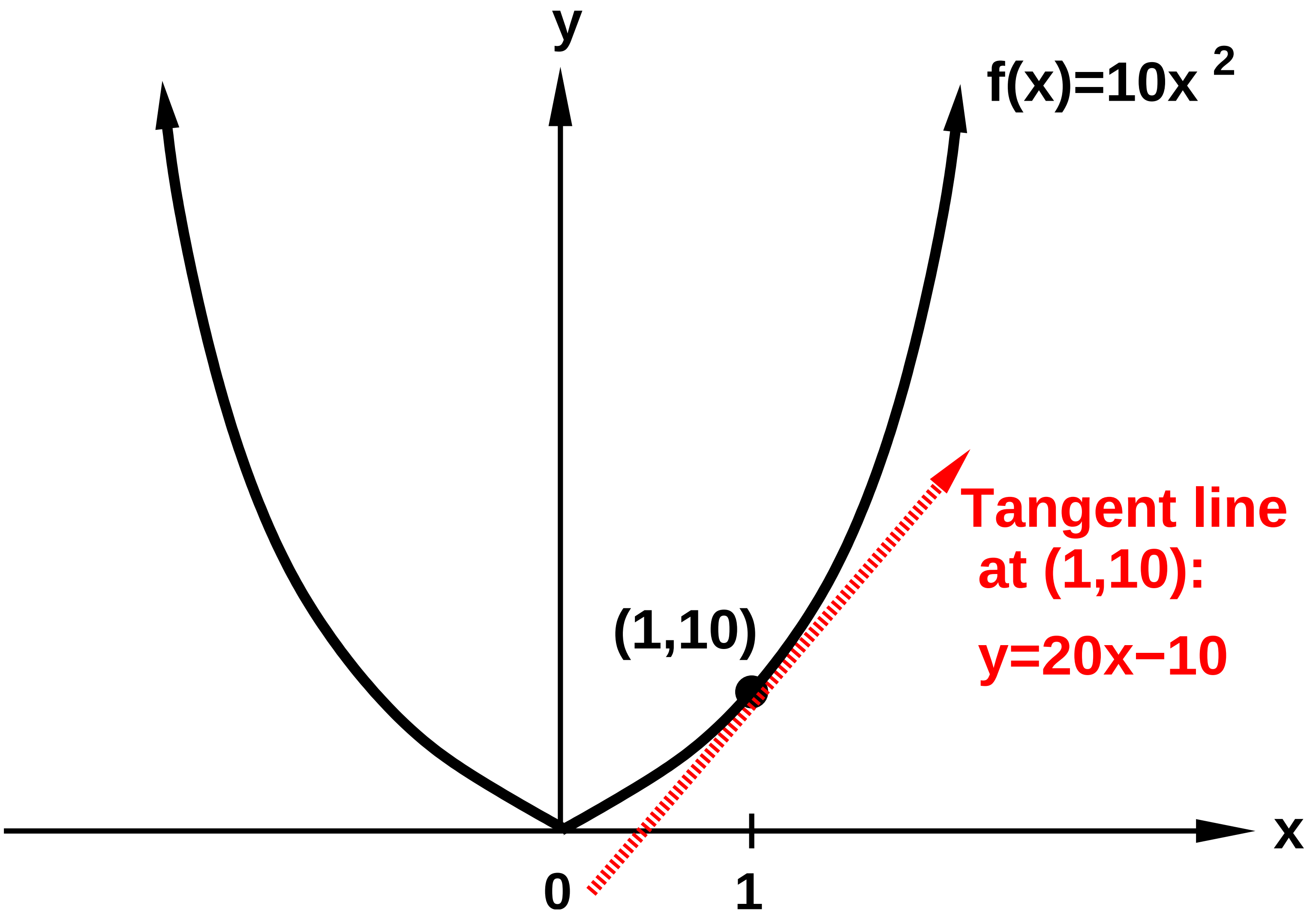
Two interpretations of the derivative

For a differentiable function f(x) and a point P = (c,f(c)):

1. f'(c) = the slope of the tangent line at P.

2. f'(c) = the rate of change of f(x) w.r.t. x when x=c.

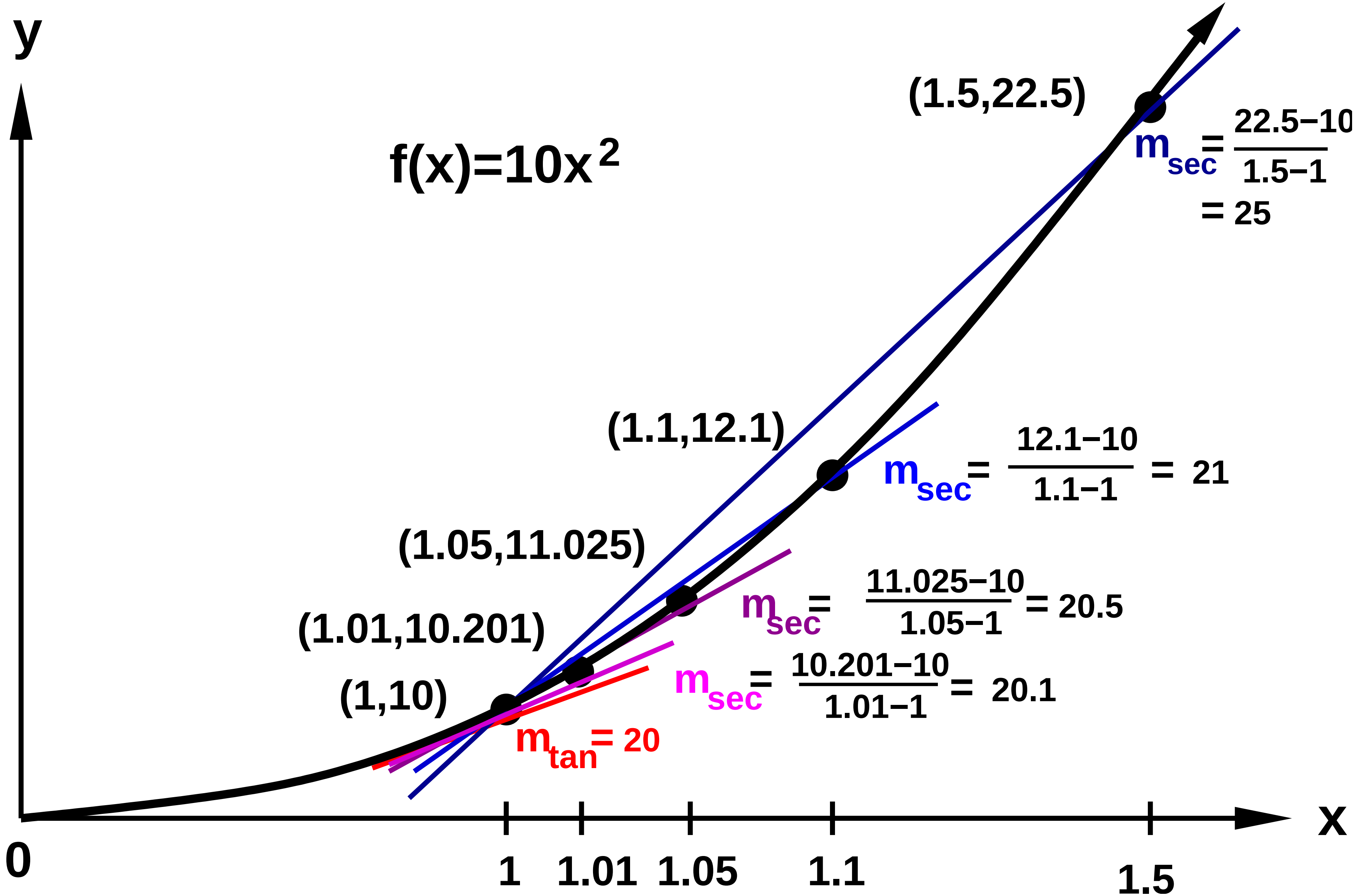
Example from last time:



f'(x) = 20x

f'(1) = 20·1 = 20 = slope of the tangent line at the point (1,10)

Example (continued):



As **h** $\longrightarrow$ **0**, $m_{sec} \longrightarrow 20 = m_{tan}$.