MATH 3283 W PROFESSIONAL PROBLEM #5 | DUE 5/7/10

FINITE AND COUNTABLY INFINITE SETS; BIJECTIONS

1. LET A, B BE FINITE SETS WHERE $\#A = m$ AND $\#B = n$, LET $f: A \longrightarrow B$ BE A FUNCTION.
   1. PROVE THAT IF f IS 1-TO-1, THEN $m \leq n$
   2. PROVE THAT IF f IS A BIJECTION, THEN $m = n$.

2. LET A, B BE FINITE SETS AND $F(A,B)$ THE SET OF ALL FUNCTIONS $f: A \to B$.
   1. SUPPOSE $A = \{a_1, a_2, a_3\}$ AND $B = \{1, 2, 3\}$. FIND $\# F(A,B)$. SHOW YOUR REASONING
   2. SUPPOSE $A = \{a_1, a_2, a_3, a_4\}$ AND $B = \{0, 1\}$. FIND $\# F(A,B)$. SHOW YOUR REASONING
   3. SUPPOSE $\#A = m$ AND $\#B = n$. FIND $\# F(A,B)$. PROVE THAT YOUR REASONING IS CORRECT

3. CONSIDER THE FUNCTION $f: \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$ DEFINED BY $f(n,m) = \frac{1}{2}(n+m-2)(n+m-1) + m$
   1. COMPUTE $f(1,1)$, $f(2,1)$, $f(1,2)$, $f(1,3)$, $f(2,2)$ AND $f(3,1)$
   2. Show that $f(n,m) = f(n', m')$ implies $(n,m) = (n', m')$ in the following cases: a) $n + m = n' + m'$. b) $n+m$ not equal to $n' + m'$, but $n = n'$ c) $n + m$ not equal to $n' + m'$, but $m = m'$.
   3. FIND PAIRS (n,m) SUCH THAT a) $f(n,m) = 13$ b) $f(n,m) = 19$, c) $f(n,m) = 28$.
   4. IT CAN BE SHOWN THAT f IS ONTO, HOW IS f RELATED TO THE BIJECTION $g: \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ DESCRIBED IN THE LECTURE NOTES?