EXERCISES - SEC 4.2 - 4.4

SEC 4.2 - 2.11, 2.12, 2.14, 2.16, 2.18, 2.20

SEC 4.3 - 3.5, 3.6

SEC 4.4 - PROOF OF THM 4.6, (2)

PROVE: IF $0 \leq x \leq 2$, $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ CONVERGES

EXAM 2 - FRIDAY, 3/26

COVERS CHAPTER 3, LECTURES AND NOTES

REVIEW - WED, 3/24 LECTURE

NO OFFICE HRS, FRIDAY, 3/12

SPECIAL OFFICE HRS, MONDAY, 3/22, 10:30 - 12 NOON, VH 4

HW FOR THIS WEEK DUE TUESDAY, 3/23

The convergence or divergence of a series can sometimes be deduced from the convergence or divergence of a closely related improper integral.

Theorem 13.3.2 The Integral Test

If f is continuous, decreasing, and positive on $[1, \infty)$, then

$$\sum_{k=1}^{\infty} f(k) \quad \text{converges} \quad \text{iff} \quad \int_1^{\infty} f(x)\,dx \quad \text{converges.}$$

PROOF. f continuous, decreasing, and positive on $[1, \infty)$ IMPLIES S_n

$$\int_1^{\infty} f(x)\,dx \quad \text{converges} \quad \text{iff} \quad \text{the sequence} \quad \left\{\int_1^n f(x)\,dx\right\} \quad \text{converges.}$$

We assume this result and base our proof on the behavior of the sequence of integrals. To visualize our argument see Figure 13.3.1.

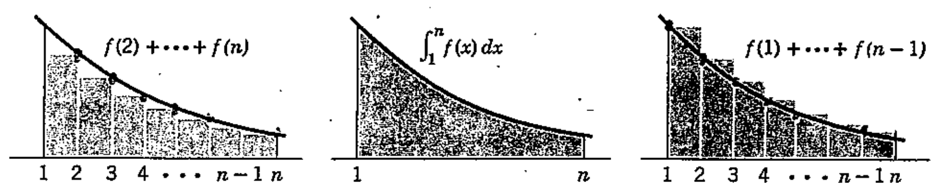

FIGURE 13.3.1

Since f decreases on the interval $[1, n]$,

$$f(2) + \cdots + f(n) \quad \text{is a lower sum for } f \text{ on } [1, n]$$

and

$$f(1) + \cdots + f(n-1) \quad \text{is an upper sum for } f \text{ on } [1, n].$$

Consequently

$$(1) \quad f(2) + \cdots + f(n) \le \int_1^n f(x)\,dx = S_n \quad \text{and} \quad \int_1^n f(x)\,dx \le f(1) + \cdots + f(n-1).$$

$\langle S_n \rangle$

$\forall n,\ S_n \le \sup S_n$

$n,\ f(2) + \cdots + f(n) \le \int_1^n f(x)\,dx$

If the sequence of integrals converges, it is bounded. By the first inequality the sequence of partial sums is bounded and the series is therefore convergent.

Suppose now that the sequence $\langle S_n \rangle$ of integrals diverges. Since f is positive, the sequence of integrals increases:

$$\int_1^n f(x)\,dx < \int_1^{n+1} f(x)\,dx.$$

Since this sequence diverges, it must be unbounded. By the second inequality, the sequence of partial sums must be unbounded and the series divergent. □

GROUP WORK 2, SECTION 11.3

Unusual Sums

In each of the following problems, determine if the sum converges, diverges, or if there is not enough information to tell:

1. $\sum_{n=1}^{\infty} \overbrace{\int_n^{n+1} \frac{1}{x^{5/3}}\,dx}^{= a_n}$

$$Q_n = a_1 + \cdots + a_n$$

$$= \int_1^2 + \int_2^3 + \int_3^4 + \cdots + \int_n^{n+1} = \int_1^{n+1} \frac{1}{x^{5/3}}\,dx$$

$$\sum_1^{\infty} = \int_1^{\infty} \frac{1}{x^{5/3}}\,dx = \frac{1}{5/3-1} = \frac{3}{2}$$ CONVERGES

2. $\sum_{n=1}^{\infty} \int_n^{n+1} x^{2/3}\,dx$

$$Q_n = \int_1^{n+1} x^{2/3}\,dx \ge \int_1^{n+1} 1\,dx = n$$

DIVERGES

3. $\sum_{n=1}^{\infty} \overbrace{\int_n^{n+\frac{1}{2}} x^{-5/3}\,dx}^{b_n =}$

$$0 \le b_n = \int_n^{n+\frac{1}{2}} \frac{1}{x^{5/3}}\,dx \le \int_n^{n+1} \frac{1}{x^{5/3}}\,dx = a_n$$

CONVERGES BY COMPARISON TEST

4. $\sum_{n=1}^{\infty} \int_n^{2n} \frac{1}{x^{5/3}}\,dx$ EXERCISE