Math 5587 Midterm I Name:____________________

1. Determine whether the following statements are true or false. No justification is required. [12 points]

   (a) The superposition principle states that if $u_1(x,y), u_2(x,y), u_3(x,y), \ldots, u_n(x,y)$ are solutions of the same PDE
   $$F(u, u_x, u_y, u_{xx}, u_{yy}, u_{xy}) = 0,$$
   then the linear combination
   $$u(x,y) = c_1u_1(x,y) + c_2u_2(x,y) + \cdots + c_nu_n(x,y),$$
   is also a solution of the same PDE, where $c_1, \ldots, c_n$ are real numbers.

   (b) Suppose u solves the heat equation $u_t - ku_{xx}$ on the infinite strip $t > 0$ and $0 < x < 1$ with Neumann boundary conditions $u_x(0,t) = 0 = u_x(1,t)$, and initial condition $u(x,0) = 2x$. Then for all $t > 0$
   $$\int_0^1 u(x,t)\,dx = 1.$$

   (c) d'Alembert's formula for the solution of the wave equation with zero initial velocity is
   $$u(x,t) = \frac{1}{2}(u(x-ct,0) + u(x+ct,0)).$$

   (d) If $f(x)$ is an odd function, then $g(x)$ defined by $g(x) = f(x)^2$ is an even function.

2. Find the solution of $u_t + u_x = 0$ satisfying $u(x, 0) = \sin(x)$. [8 points]

3. Solve the wave equation $u_{tt} - c^2 u_{xx} = 0$ on the entire real line $-\infty < x < \infty$ with initial position $u(x, 0) = e^{\frac{1}{x^2+1}}$ and initial velocity $u_t(x, 0) = x$. Simplify your expression for u as much as possible. [10 points]

4. Find the solution of $u_t + \sqrt{x} u_x = 0$ on the domain $x > 0$ and $t > 0$ that satisfies $u(x, 0) = x$. [10 points]

5. Solve the heat equation $u_t - u_{xx} = 0$ on the entire real line $-\infty < x < \infty$ with initial condition $u(x,0) = \cos(x)$ without using the fundamental solution of the heat equation. Use your answer to deduce the value of the integral

$$\int_{-\infty}^{\infty} \cos(x) e^{-x^2/4t}\, dx.$$

[Hint: Look for a solution of the form $u(x,t) = g(t)\cos(x)$.] [10 points]