Math 5587 – Homework 12 (Due Thursday Dec 15)

1. Find the Green's function for the tilted half-space
$$D = \{(x, y, z) \,:\, ax + by + cz > 0\}.$$
[Hint: Use the Green's function for the halfspace $\{z > 0\}$ and a change of variables.]

2. Find the Green's function for the half ball
$$D = \{(x, y, z) \,:\, x^2 + y^2 + z^2 < a^2 \;\text{ and }\; z > 0\}.$$
[Hint: Reflect the solution for the whole ball across the plane $z = 0$.]

3. Consider the two dimensional disk
$$D = \{(x, y) \,:\, x^2 + y^2 < a^2\}.$$
Show that the Green's function for the disk is
$$G(\mathbf{x}, \mathbf{x}_0) = \frac{1}{2\pi} \log(\|\mathbf{x} - \mathbf{x}_0\|) - \frac{1}{2\pi} \log\left(\frac{\|\mathbf{x}_0\|}{a} \|\mathbf{x} - \mathbf{x}_0^*\|\right).$$
where $\mathbf{x}_0^* = \frac{a^2 \mathbf{x}_0}{\|\mathbf{x}_0\|^2}$.

4. Use problem 3 to recover the two dimensional version of Poisson's formula for the ball that we derived in class using separation of variables.