Number theory exercises 06

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Due Wed, 14 Dec 2011, preferably as PDF emailed to me.

[number theory 06.1] State and prove the *product formula* for $\mathbb{F}_q(x)$.

[number theory 06.2] Show that for irrational $\theta \in \mathbb{R}$, the collection $m + n\theta$ with $m, n \in \mathbb{Z}$ is *dense* in $\mathbb{R}$.

[number theory 06.3] State and prove the analogue of Ostrowski's theorem for absolute values on $\mathbb{F}_q(x)$.

[number theory 06.4] Prove that $\mathbb{A}/k$ is compact for $k = \mathbb{F}_q(x)$.

[number theory 06.5] Prove carefully that for α in a number field k, if $\alpha \in \mathfrak{o}_v$ for every non-archimedean v, then $\alpha \in \mathfrak{o}$. [*Locally integral everywhere implies globally integral.*]

[number theory 06.6] * *[Starred problems are optional]* Let θ be a complex cube root of 2. Show that the collection of $a + b\theta + c\theta^2$ with $a, b, c \in \mathbb{Z}$ is dense in $\mathbb{C}$.