1. Lily the ladybug, Bob the beetle, and Sid the Spider all live happily in the $xy$ plane. At time $t$ Bobby’s position is given by the parametric equation

$$b(t) = \begin{pmatrix} \cos(\pi t) \\ \sin(\pi t) \end{pmatrix}$$

Simultaneously, Lily’s position is given by

$$l(t) = \begin{pmatrix} t - 1 \\ t \end{pmatrix}$$

(a) Do the path of Bob and Lily ever cross paths? Do Bob and Lily ever meet?

(b) At time $t = 0$ Sid is located at the point $(-3, -2)$ and begins running in a straight line towards the point $(-1, 0)$, arriving at time $t = 1$. Find a parametric equation $s(t)$ for Sid’s path.

(c) Does Sid cross paths with Bob or Lily? Does Sid ever run into Bob or Lily?

2. Use function $f(x, y) = x^2y + y^2$ for the following exercises.

(a) Using the definition of the partial derivative, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

(b) Double check your answer by computing the $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ using the derivative rules. Keeping in mind that when you compute $\frac{\partial f}{\partial x}$ you view $y$ as a constant.

(c) Find the $x = c$ and $y = c$ cross sections of the graph of $f(x, y)$ for a constant $c$. Both cross sections are the graphs of function $\mathbb{R} \rightarrow \mathbb{R}$, find the derivatives of both functions.

(d) The $y = x$ cross section of $f(x, y)$ is the graph of some function $g(x)$. Find $g(x)$.

3. A function $f(x, y)$ is harmonic if it satisfies the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Show that $f(x, y) = x^3 - 3xy^2$ is harmonic.

4. The heat equation is: $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$. Show that $u(x, t) = e^{-k^2t} \sin(x)$ is a solution of the heat equation.

5. Find the matrix of partial derivatives of the function

$$F(x, y, z) = (ze^{x^2+y^2} + xy, \cos(x^3y^2z^4))$$

6. Let $f(x, y) = x^2 + \frac{1}{2}y^2 - 2x$ Find a point on the graph $z = f(x, y)$ where the tangent plane is horizontal.

7. Let $f(x, y) = \frac{x}{y} + \frac{y}{x}$. Using a linear approximation about the point $(1/2, 1/4)$, estimate the value of $f(0.48, 3)$. 