Unipotent Representations and Unitarity

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Abstract: A representation $\pi : G \to Aut(V)$ of a group G is called unitary if V has a positive hermitian form such that G acts by unitary operators, i.e. $< \pi(g)v, \pi(g)w > = < v, w >$ for all $g \in G,\ v, w \in V$. An important problem in representation theory, which has applications in analysis, mathematical physics and number theory is the classification of the unitary irreducible representations of a reductive group. There are operations, unitary induction, derived functor construction and complementary series which take representations of a Levi component M of G to representations of G, and which preserve unitarity. The first step is then to determine which unitary representations of G are **basic**, i.e. they do not arise by the above operations from unitary representations of proper Levi components. **Unipotent** representations are irreducible representations of G which have properties closely connected to nilpotent orbits in the Lie algebra. Basic representations are conjectured to be unipotent.

In this talk I will review some constructions and properties of unipotent representations, and describe their role in the classification of unitary representations.