HYPERGEOMETRIC SERIES, AUTOMORPHIC FORMS, AND MOCK THETA FUNCTION

In my talk I will describe some of the connections between hypergeometric series and automorphic forms. The literature on examples of hypergeometric series that are related to modular forms is extensive, and the pursuit of further of these and their interpretation is an active area of research due to their applications to many areas of mathematics and to physics. However, the proofs of these scattered results fall far short of a comprehensive theory to describe the interplay between hypergeometric series and automorphic forms. The situation is further complicated by the mock theta functions, a collection of 22 q-series defined by Ramanujan in his last letter to Hardy. Though they resemble modular q-series, these functions do not arise as minor modifications of the Fourier expansions of modular forms. Recently, much light has been shed on the nature of Ramanujan's mock theta functions and it is now known that these functions are the holomorphic parts of weight $\frac{1}{2}$ weak Maass forms, and a clearer picture is beginning to emerge of which modular forms and Maass forms arise from basic hypergeometric series. I will describe part of those results.