

Homework #3 for MATH 8301: Manifolds and Topology

September 19, 2017

Due Date: Monday 25 September in class.

1. Prove that the geometric realization of a simplicial complex is compact if and only if it has finitely many vertices.
2. Prove that the geometric realization of a simplicial complex is connected if and only if for every pair of vertices, v and w , there exists a string of vertices $v = v_0, v_1, \dots, v_n = w$ so that each edge $e_m = \{v_{m-1}, v_m\}$ lies in the simplicial complex, for $m = 1, \dots, n$.
3. Consider the quotient space of the hexagon, where the edges are identified according to the string $abcab^{-1}c$. What surface does this describe? You're welcome to solve this problem by drawing pictures, using cut-and-paste techniques.