Homework #4 for MATH 5345H: Introduction to Topology

October 5, 2018

Due Date: Friday 12 October in class.

1. Show that the collection $\{(a,b) \times (c,d) \mid a,b,c,d \in \mathbb{Q},\ a < b, c < d\}$ is a countable basis for the product topology on $\mathbb{R}^2$.
2. Let X be an ordered set, and give it the order topology. Show that the closure $\overline{(a,b)}$ of the open interval (a,b) is contained in the closed interval $[a,b]$; that is, $\overline{(a,b)} \subseteq [a,b]$. Give a condition (i.e., a properties of the order) in which they are equal for every $a < b$.
3. Show that the dictionary order topology on $\mathbb{R} \times \mathbb{R}$ is the same as the product topology on $\mathbb{R}_d \times \mathbb{R}$, where $\mathbb{R}_d$ denotes $\mathbb{R}$ with the discrete topology.
4. Let $A = \{\frac{1}{n} \mid n > 0\} \subseteq \mathbb{R}$. Prove your answer for the following questions:
   (a) If we give $\mathbb{R}$ the standard topology, compute the set of limit points of A.
   (b) In contrast, if we give $\mathbb{R}$ the finite complement topology, compute the set of limit points of A.
   (c) Generally, let X be a set, and let T and T' be two topologies on X where T is finer than T' (i.e., $T' \subseteq T$). If $A \subseteq X$ is a subset, and $a \in X$ is a limit point of A with respect to T, show that a is also a limit point with respect to T'.