

Solution 1

$$\begin{aligned}y &= \frac{5e^x}{1+3e^x} \\y(1+3e^x) &= 5e^x \\y+3ye^x &= 5e^x \\y &= 5e^x - 3ye^x \\y &= e^x(5-3y) \\\frac{y}{5-3y} &= e^x \\\ln \frac{y}{5-3y} &= \ln e^x \\x &= \ln \frac{y}{5-3y}\end{aligned}$$

Solution 2

$$\begin{aligned}y &= \frac{5e^x}{1+3e^x} \\\frac{1}{y} &= \frac{1+3e^x}{5e^x} \\\frac{1}{y} &= \frac{1}{5e^x} + \frac{3}{5} \\\frac{5-3y}{5y} &= \frac{1}{5e^x} \\\frac{5-3y}{y} &= \frac{1}{e^x} \\\frac{y}{5-3y} &= e^x \\\ln \frac{y}{5-3y} &= \ln e^x \\x &= \ln \frac{y}{5-3y}\end{aligned}$$