OPTIMIZATION WORKSHEET SOLUTIONS

MATH 1271

(1) We don't have a constraint on the dimensions (other than that the pitch can't be square, which isn't possible anyway), so we can just pick the biggest and smallest lengths and widths. Then the largest possible pitch is 130×101 and the smalelst is 110×50.

(2) Suppose the farmer builds a square enclosure and a triangular enclosure having side lengths x and y, respectively. Then we have the constraint on the perimeter that $4x + 3y = 10$, so $x = \frac{10-3y}{4}$. Then the area is $A = x^2 + \frac{\sqrt{3}y^2}{4}$. We can then substitute for x to have the area as a function of y:

$$A(y) = \frac{(10-3y)^2}{16} + \frac{\sqrt{3}y^2}{4}.$$

Then

$$A'(y) = \frac{(10-3y)(-3)}{8} + \frac{\sqrt{3}y}{2} = -\frac{30}{8} + \frac{9y}{8} + \frac{\sqrt{3}y}{2}.$$

We solve $A'(y) = 0$ and get $y \approx 1.88$. Noting that $0 \leq y \leq \frac{10}{3}$ we then have

y	$A'(y)$
$0 < y < 1.88$	< 0
$1.88 < y < \frac{10}{3}$	$x > 0$

Then the maximum area occurs either when $y = 0$ or $y = 10/3$ since the area decreases for $0 < y < 1.88$ and increases for $y > 1.88$. $A(0) = \frac{25}{4}$, while $A(10/3) \approx 4.8$. Thus the largest possible enclosure is a square enclosure with sides of length $10/4$. The smallest possible arrangement is a triangle with sides of length 1.88 and a square with sides of length $\dfrac{10-3(1.88)}{4}$.

(3) We have that $s(t) = 10 + 3t - 4.9t^2$, so $s'(t) = v(t) = 3 - 9.8t$. Then $v'(t) = -9.8$, so the velocity is always decreasing. Therefore the maximal velocity occurs when the ball is thrown, i.e. $v(0) = 3$. It is worth noting that the *speed* (i.e. the absolute value of the velocity) is increasing (due to gravity) and is greatest when the ball hits the ground. Using the quadratic formula we can see that $s(1.15) = 0$ and $v(1.15) = -8.32$. Consequently, the maximal speed is 8.32.