This is an open book, library, notes, web, take-home exam, but you are not to collaborate. Your instructor is the only human source you are allowed to consult. Be sure to cite all outside sources you use. Problem parts marked with * are worth 10 points, and all others are worth 5 points.

1. (a) Let ϕ be the ring homomorphism $\mathbb{Z}[x,y] \to \mathbb{Z}[t]$ that sends $f(x,y) \mapsto f(t^6,t^8)$. Find, with proof, a generating set of minimum cardinality for the ideal $I = \ker\phi$ in $\mathbb{Z}[x,y]$.
   (b) A prime ideal is *minimal* if it does not contain any smaller prime ideal. Find, with proof, all of the minimal prime ideals in the ring $\mathbb{Z}[x_1,x_2]/\langle x_1x_2\rangle$.
   (c) Show that the ideal in $\mathbb{Z}[x]$ generated by x^2+1 and 11 is maximal.

2. Prove or disprove the following statements about a commutative ring R with 1, always interpreting "subring" to mean "subring with 1":
   (a) If I is an ideal in R and R/I is a domain, then R is also a domain.
   (b) If R is a UFD, then any subring of R is a UFD.
   (c) If R is a Euclidean domain, then any subring of R is a Euclidean domain.
   (d) *If R is a UFD and I is a prime ideal of R, then R/I is a UFD.
   (e) *If R is a Noetherian ring, then any subring of R is a Noetherian ring.

3. (a) *Prove x^5+y^7+1 is irreducible in $\mathbb{Q}[x,y]$.
   (b) *Prove $x^8+y^4+z^6$ is reducible in $\mathbb{F}_p[x,y,z]$ when $p=2$, but irreducible in $\mathbb{F}_p[x,y,z]$ for all odd primes p.

4. *Grant that the ring $\mathbb{Z}[i]$ of Gaussian integers is Euclidean, thus is a PID. Observe that $(2+i)(2-i)=5$. How many isomorphism classes of $\mathbb{Z}[i]$-modules with exactly 5 elements are there?

5. Let V be a vector space over $\mathbb{C}$, and let $T\colon V \to V$ be a linear mapping. We regard V as a module for $\mathbb{C}[x]$ by letting x act in the same way as T. Suppose that V is the direct sum of two cyclic $\mathbb{C}[x]$-modules, whose annihilators are $(x^2-4)^2$ and x^3-2x^2, so $V \cong \mathbb{C}[x]/(x^2-4)^2 \oplus \mathbb{C}[x]/(x^3-2x^2)$.
   (a) *Calculate both the minimal and characteristic polynomials of T.
   (b) *Calculate the Jordan canonical form of T.

Problem	1	2	3	4	5	Total
Points	15	35	20	10	20	100
Score						