Math 5385 Project - Spring 2018

Task. Learn a new theorem related to the course material and communicate it in both verbal and written form.

Minimum requirements.

- Each student is expected to learn a different theorem.
- The written document must introduce, correctly state, and prove the theorem. It should also include at least one interesting example illustrating the theorem. The article should be as self-contained as possible. The target audience for this paper is the other students enrolled in the course. The new document must be typed, be at most eight pages in length (with one inch margins and a 12pt font), and be available in PDF format.
- The verbal presentation must introduce and state the theorem. It should also include at least one example illustrating the theorem.

Assessment and deadlines. Project grades will be computed as follows.

due date	element	weight
02-05-2018	topic by email (5pm)	2%
02-19-2018	outline	8%
03-26-2018	rough draft	10%
04-02-2018	feedback	10%
04-23-2018	presentations begin	30%
04-25-2018	final paper	30%
05-02-2018	feedback on presentations	10%

Comments. By design, this assignment is open-ended. Students are strongly encouraged to compute many examples. Students are encouraged to formulate, test, and prove their own conjectures. The course webpage contains links to suggestions on how to present on mathematics (see Halmos on writing, Kleiman on writing, and Halmos on talking).

Potential topics. Roughly speaking, you could choose to study about 10-15 pages in any of the books in the references. Here are some more specific ideas. It is important to note that these topics generally contain far more material than would be appropriate for your project. That means that your job is to identify the results that you think are most interesting and then present them in an accessible and condensed manner. This is much harder than it may sound at first!

- algebraic statistics: [St2] §5 (Theorem 5.3); [St3] §8 (Theorem 8.14)
- Alexander duality: [MS] §5 (Theorem 5.24); [HH] §8.1 (Theorem 8.1.6)
- automatic theorem proving: [IVA] §6.4 (Proposition 6.5.8)
- Barvinok's theorem: [MS] §12.4 (Theorem 12.18)
- Bernstein's theorem: [UAG] §7.5 (Theorem 7.5.4); [St3] §3 (Theorem 3.2)
- Brion's formula: [MS] §12.3 (Theorem 12.13)
- combinatorial Nullstellensatz: [TV] §9.1 (Theorem 9.2)
- computation in local rings: [UAG] §4 (Theorem 4.2.2, Theorem 4.4.2); [GP] §6 (Theorem 6.2.6)
- Fröberg's theorem: [HH] §9.2 (Theorem 9.2.3)
- generic initial ideals: [E] §15.9 (Theorem 15.18); [HH] §4 (Theorem 4.1.2)

- Grassmannians: [H] §11 (Proposition 11.30)
- Hilbert schemes: [MS] §18.2 (Theorem 18.7)
- Hilbert syzygy theorem: [UAG] §6 (Theorem 6.2.1) [E] §15.5 (Theorem 15.10)
- integer programming: [UAG] §8.1-2 (Theorem 8.1.11); [St2] §5 (Theorem 5.5)
- invariant theory of finite groups: [IVA] §7 (Theorem 7.3.5); [St1] §2 (Theorem 2.1.3)
- Ishida complex: [MS] §13.3 (Theorem 13.24)
- Koszul complex: [E] §17.1 (Theorem 17.1); [GP] §7 (Theorem 7.6.14)
- linear partial differential equations: [St3] §10 (Theorem 10.3)
- multigraded Hilbert series: [MS] §8.3 (Theorem 8.20)
- Noether normalization: [E] §13.1 (Theorem 13.3)
- Puiseux series: [E] §13.3 (Corollary 13.15); [St3] §1.4 (Theorem 1.7)
- Quillen–Suslin theorem: [UAG] §5.1 (Theorem 5.1.8); [BG] §8 (Theorem 8.5)
- resolutions of monomial ideals: [MS] §3.5 (Theorem 3.17); [HH] §7 (Theorem 7.1.1)
- resultants: [UAG] §3 (Theorem 3.2.3); [St3] §4 (Theorem 4.6)
- SAGBI basis: [St2] §11 (Theorem 11.4); [MS] §14.3 (Theorem 14.11)
- saturations of affine semigroup rings: [MS] §7.3 (Proposition 7.25)
- Stickelberger's theorem: [UAG] §2 (Theorem 2.4.5); [St3] §2.3 (Theorem 2.6)
- straightening laws: [St1] §3 (Theorem 3.1.7, Theorem 3.2.1)
- sums of squares: [St3] §7 (Theorem 7.3)
- tropical hypersurfaces: [St3] §9 (Theorem 9.17)
- triangulations and toric ideals: [St2] §8 (Theorem 8.3); [BG] §7 (Theorem 7.18)
- universal Gröbner bases: [St2] §1 (Theorem 1.4), §7 (Theorem 7.1)

References

[BG] Winfried Bruns and Joseph Gubeladze, *Polytopes, rings, and K-theory, Monographs in Mathematics*. Springer, 2009, ISBN 978-0-387-76355-2.

[IVA] David A. Cox, John B. Little, and Don O'Shea, *Ideals, Varieties, and Algorithms*, third edition, Springer, 2007, ISBN 978-0-387-35650-1.

[UAG] David A. Cox, John B. Little, and Don O'Shea, *Using Algebraic Geometry*, GTM 185, Springer, 2005, ISBN 978-0-387-20733-9.

[E] David Eisenbud, *Commutative algebra with a view towards algebraic geometry*, GTM 150. Springer, 1995, ISBN 0-387-94268-8.

[GP] Gert-Martin Greuel and Gerhard Pfister, *A Singular introduction to commutative algebra*, 2nd edition, Springer, 2008, ISBN 978-3-540-73541-0.

[H] Brendan Hassett, *Introduction to Algebraic Geometry*, Cambridge University Press, 2007, ISBN 978-0-521-69141-3.

[HH] Jürgen Herzog and Takayuki Hibi, *Monomial ideals*, GTM 260. Springer, 2011, ISBN 978-0-85729-105-9.

[MS] Ezra Miller and Bernd Sturmfels, *Combinatorial commutative algebra*, GTM 227. Springer, 2005, ISBN 0-387-22356-8.

[St1] Bernd Sturmfels, *Algorithms in invariant theory*, Texts and Monographs in Symbolic Computation. Springer, 1993, ISBN 3-211-82445-6.

[St2] Bernd Sturmfels, *Gröbner bases and convex polytopes*, University Lecture Series 8. American Mathematical Society, 1996, ISBN 0-8218-0487-1.

[St3] Bernd Sturmfels, *Solving systems of polynomial equations*, CBMS 97, American Mathematical Society, 2002, ISBN 0-8218-3251-4.

[TV] Terence Tao and Van H. Vu, *Additive combinatorics*, Cambridge University Press, 2010, ISBN 978-0-521-13656-3.