Name: ________________________

Problem Set 12
Math 4281, Spring 2014
Due: Wednesday, April 23

Cosets

1. Suppose $G = \langle c \rangle$ is a cyclic group of order 8. List the cosets of $\langle c^4 \rangle$.

Normal subgroups and quotient groups

2. Show that H is a normal subgroup of G, but K is not a normal subgroup of G, where
$$G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} \right\},$$
$$H = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \right\}, \quad \text{and}$$
$$K = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}.$$

3. Prove that $Z = \{a \in G \mid ax = xa \text{ for all } x \in G\}$ is a normal subgroup of G. (This is called the **center** of G.)

4. Let G be a group with identity element e. Suppose that $H, K \subseteq G$ are subgroups of orders 5 and 8, respectively. Prove that $H \cap K = \{e\}$.

5.
   a. Prove that a group G of even order has an element of order 2. (Hint: If $a \neq e$, then a has order 2 if and only if $a = a^{-1}$.)
   b. Suppose m is odd, $|G| = 2m$, and G is abelian. Prove G has precisely one element of order 2. (Hint: If there were two, they would provide a Klein four-group.)
   c. Prove that if G has exactly one element of order 2, then it must be in the center of G.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________________