Name: _____

Problem Set 1
Math 4281, Spring 2014
Due: Wednesday, January 29

Complete the following items, staple this page to the front of your work, and turn your assignment in class on Wednesday, January 29.

1. Carefully read the entire course website. Send an email to your instructor containing:
   - `Math4281` in the subject line;
   - a review of the course website (What did you like? What would you change? List any typos you discovered, etc.); and
   - an acknowledgement that you understand the policies and procedures for this course.

Review

2. Negate the following sentences; in each case, indicate whether the original sentence or its negation is a true statement. Be sure to move the "not" through all the quantifiers.
   (a) For every integer $n \geq 2$, the number $2^n - 1$ is prime.
   (b) There exists a real number M so that for all real numbers t, $|\sin t| \leq M$.
   (c) For every real number $x > 0$, there exists a real number $y > 0$ so that $xy > 1$.

3. An integer n is called *wonderful* provided that whenever n divides ab, then n divides a or n divides b. (Here, a and b must also be integers.)
   (a) Complete this sentence: n is *not wonderful* if...
   (b) Decide whether the integer 6 is wonderful. Explain carefully.

4. Explain your answers.
   (a) Define a function $f : \mathbb{N} \to \mathbb{N}$ that is one-to-one but not onto.
   (b) Define a function $f : \mathbb{N} \to \mathbb{N}$ that is onto but not one-to-one.

Preliminaries

5. Prove the following with mathematical induction:
   (a) The sum of the first n positive integers is $\frac{n(n+1)}{2}$.
   (b) For an integer $n \geq 1$, $n^3 - n$ is divisible by 3.
   (c) For any positive integer n, one of $n, n+1, n+2$ must be divisible by 3.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: _____