Working primarily with n-dim'l real space : notated $\mathbb{R}^n$.

not going to construct $\mathbb{R}$, but do use various properties — see Section 0.5

(do this in Honors Analysis for example)

e.g. every non-empty subset ~~has~~ of $\mathbb{R}$ with upper bound has least upper bound.

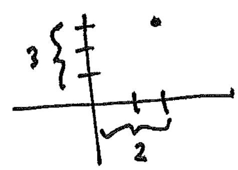
i.e. smallest

want to be precise and understand why, but also want to get somewhere — so take a few results as granted...

Points versus vectors : Book denotes points as column of n-numbers in $\mathbb{R}^n$ $\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

geometric intuition : fixing n mutually perp. directions and x_i's represent distance in i^{th} direction.

e.g. $\mathbb{R}^2$: $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ represents point



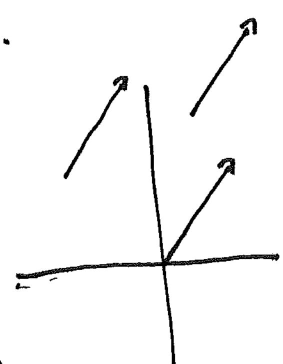
vector is described by same data: now with square brackets $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} =: \vec{x}$ in $\mathbb{R}^n$

so what's the difference? vector represents distance in given direction w/o fixed coordinate system.

Geometrically:

Draw

as rep'n of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ but all of these are ways of plopping $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ onto fixed coord. system.



Easy to move back and forth between two languages. E.g. point $\rightsquigarrow$ vector by thinking of line joining origin to point.

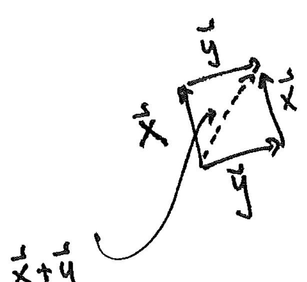
We find language of vectors is preferable (has better geom. intuition) for describing basic operations of scalar mult. + addition.

<u>Scalar mult</u>: $c \in \mathbb{R}$, $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$ then $c \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \overset{\text{def}}{=} \begin{bmatrix} cx_1 \\ \vdots \\ cx_n \end{bmatrix}$

geometrically, stretching vector by c in all directions.
(if $c<0$, then also reversing directions) Do example.

<u>addition</u>: component-wise addition: $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \overset{\text{def}}{=} \begin{bmatrix} x_1+y_1 \\ \vdots \\ x_n+y_n \end{bmatrix}$

geometrically, placing vectors $\vec{x}, \vec{y}$ in succession, tail to head.
(if we didn't already know $x_i + y_i = y_i + x_i$, picture confirms this: commutativity prop.



) Do example.

<u>Remark</u>: combine two operations to get subtraction: $\vec{x} + (-1)\cdot\vec{y}$

<u>linear combination</u>: any vector of form $c_1\vec{v}_1 + \cdots + c_k\vec{v}_k$
of $\vec{v}_1, \dots, \vec{v}_k \in \mathbb{R}^n$ with $c_i \in \mathbb{R}$. $i = 1, \dots, k$.
(i.e. all possible operations on vectors $\vec{v}_1, \dots, \vec{v}_k$)

<u>Question</u>: Which subsets of $\mathbb{R}^n$ are closed under linear combination?
Such a subset is called a "subspace".

Short Answer: Most fundamental subsets: lines, planes, etc. passing through origin.

Do examples (subspace generated by vector, $\mathbb{R}^n$ itself, sol'ns to eqn.), non-examples (e.g. unit circle in $\mathbb{R}^2$)

LECTURE 1 STOPPED HERE!

Question 2: Our geometric intuition for $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ presumes choice of mutually perpendicular vectors

Call them $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, ... $\vec{e}_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$.

Any vector in $\mathbb{R}^n$ is a linear combination of $\vec{e}_1, \dots, \vec{e}_n$ and no smaller subset of $\vec{e}_i$ have that lin. combs. give all vectors in $\mathbb{R}^n$

Call this a "basis" of space $\mathbb{R}^n$

Talk much more about this later, but for now I'll use this terminology for $\{\vec{e}_1, \dots, \vec{e}_n\}$. "standard basis"

At last, very interesting question: How to find bases for a given subset of $\mathbb{R}^n$? Even more subtle: How to find good bases for one's purposes? ("good" depends on context.)

Central idea of differential calculus:

approximate (smooth) functions locally by lines.

Generalize this to $\mathbb{R}^n$. First think about how we defined lines in one-variable calculus.