MATH 4567, SPRING 2018
HOMEWORK PROBLEMS No.1
Due on February 5 (Monday)

Problem 1. Problem 1 on page 194 of your textbook.

Problem 2. In the space $L^2[0, c]$, where $c > 0$, find the angle θ between the functions $f_1(x) = x$ and $f_2(x) = x^2$. (Use arccos function). Does θ depend on the parameter c?

Problem 3. Find the constants $a, b, c \in \mathbf{R}$, $a, b > 0$, such that the functions

$$f_1(x) = ax, \quad f_2(x) = bx + c$$

form an orthonormal system in $L^2[0, 1]$.

Problem 4. Find the best approximation in the mean of the function $f(x) = 1$ in $L^2[0, 1]$ by a linear combination of the functions f_1, f_2 from problem 3.

Problem 5. Find the best approximation g in the mean on the interval $0 \le x \le \pi$ for the function $f(x) = 1$ using linear combinations of

$$f_1(x) = \sin x, \quad f_2(x) = \sin 3x.$$

Then evaluate the error of approximation, that is, $\|f - g\|$ in $L^2[0, \pi]$.

Problem 6. Find the best approximation g of the function $f(x) = \cos^3 x$ in $L^2[0, 2\pi]$ by a linear combination of the functions

$$f_1(x) = \frac{1}{\sqrt{\pi}} \cos x, \quad f_2(x) = \frac{1}{\sqrt{\pi}} \sin x.$$

Then find the L^2-distance from f to $L_2[f_1, f_2]$ (i.e., compute $\|f - g\|$).

Problem 7. On the interval $[-\pi, \pi]$ find the Fourier series
a) for the function f in Problem No. 1 on page 18;
b) for the function $f(x) = |x|$;
c) for the function $f(x) = \max\{x, 0\}$.