

MATH 4567, SPRING 2015
HOMEWORK PROBLEMS No. 3
Due on March 30

Problem 1. Using elementary arguments, solve the boundary value problem

$$\begin{aligned}u_{xx} &= -y^2 \cos x, & u &= u(x, y), \quad 0 \leq x \leq \pi, \\u(0, y) &= y^2, \\u(\pi, y) &= \pi \sin y - y^2.\end{aligned}$$

Problem 2. a) Solve the temperature problem:

$$\begin{aligned}u_t &= k u_{xx}, & u &= u(x, t), \quad 0 < x < 1, \quad t > 0 \quad (k > 0 \text{ parameter}) \\u_x(0, t) &= u_x(1, t) = 0, \\u(x, 0) &= \frac{1}{2} x^2.\end{aligned}$$

Your answer will have the form of an infinite functional series.

b) Write down separately the first 4 terms of that functional series.

Problem 3. a) Solve the boundary value problem:

$$\begin{aligned}u_{tt} &= \frac{1}{2} u_{xx}, & u &= u(x, t), \quad 0 < x < 1, \quad t > 0 \\u(0, t) &= u(1, t) = 0, \\u_t(x, 0) &= 0, \\u(x, 0) &= x(1 - x).\end{aligned}$$

Your answer will have the form of an infinite functional series.

b) Write down separately the first 4 terms of that functional series and evaluate this partial sum at time $t = 1$ for the point $x = \frac{1}{2}$.

Problem 4. Solve directly for the eigenvalues and normalized eigenfunctions:

- a) No. 1 on page 225;
- b) No. 2 on page 225;
- c) No. 3 on page 225.

Problem 5. a) Given parameters $c > 0$ and $\beta > 0$, show that the Sturm-Liouville boundary value problem

$$\begin{aligned}y'' + \lambda y &= 0, & y &= y(x), \quad 0 \leq x \leq c, \\y'(0) &= \beta y(0), \\y'(c) &= \beta y(c),\end{aligned}$$

has exactly one negative eigenvalue λ_0 and that this eigenvalue is independent on $c > 0$. Find λ_0 and an associated eigenfunction $y_0(x)$.

b) Determine whether or not $\lambda = 0$ is an eigenvalue. If yes, find an associated eigenfunction.