MATH 4567, SPRING 2015
HOMEWORK PROBLEMS No.1
Due on February 9 (Monday)

Problem 1. Problem 1 on page 194 of your textbook.

Problem 2. In the space $L^2[0,c]$, where $c>0$, find the angle θ between the functions $f_1(x)=x$ and $f_2(x)=x^2$. (Use arccos function). Does θ depend on the parameter c?

Problem 3. Find the constants $a,b,c \in \mathbf{R}$, $a,b>0$, such that the functions

$$f_1(x)=ax, \quad f_2(x)=bx+c$$

form an orthonormal system in $L^2[0,1]$.

Problem 4. Find the best approximation in the mean of the function $f(x)=1$ in $L^2[0,1]$ by a linear combination of the functions f_1, f_2 from problem 3.

Problem 5. Find the best approximation g of the function $f(x)=\cos^3 x$ in $L^2[0,2\pi]$ by a linear combination of the functions

$$f_1(x)=\frac{1}{\sqrt{\pi}}\cos x, \quad f_2(x)=\frac{1}{\sqrt{\pi}}\sin x.$$

Then find the L^2-distance from f to $L_2[f_1,f_2]$ (i.e., compute $\|f-g\|$).

Problem 6. Prove Cauchy (called also Schwarz) inequality by using the hint given in problem 5, page 194, in your textbook.