MATH 4567, FALL 2014
HOMEWORK PROBLEMS No. 3
Due on March 31

Problem 1. Using elementary arguments, solve the boundary value problem

$$\begin{aligned} &u_{xx} = -y^2 \cos x, \qquad u = u(x,y),\ 0 \le x \le \pi, \\ &u(0,y) = y^2, \\ &u(\pi,y) = \pi \sin y - y^2. \end{aligned}$$

Problem 2. *a*) Solve the temperature problem:

$$\begin{aligned} &u_t = k\, u_{xx}, \qquad u = u(x,t),\ 0 < x < 1,\ t > 0 \ \ (k > 0 \text{ parameter}) \\ &u_x(0,t) = u_x(1,t) = 0, \\ &u(x,0) = \tfrac{1}{2}\, x^2. \end{aligned}$$

Your answer will have the form of an infinite functional series.
b) Write down separately the first 4 terms of that functional series.

Problem 3. *a*) Solve the boundary value problem:

$$\begin{aligned} &u_{tt} = \tfrac{1}{2}\, u_{xx}, \qquad u = u(x,t),\ 0 < x < 1,\ t > 0 \\ &u(0,t) = u(1,t) = 0, \\ &u_t(x,0) = 0, \\ &u(x,0) = x(1-x). \end{aligned}$$

Your answer will have the form of an infinite functional series.
b) Write down separately the first 4 terms of that functional series and evaluate this partial sum at time $t = 1$ for the point $x = \frac{1}{2}$.

Problem 4. Solve directly for the eigenvalues and normalized eigenfunctions:

a) No. 1 on page 225;
b) No. 2 on page 225;
c) No. 3 on page 225.

Problem 5. *a*) Given parameters $c > 0$ and $\beta > 0$, show that the Sturm-Liouville boundary value problem

$$\begin{aligned} &y'' + \lambda y = 0, \qquad y = y(x),\ 0 \le x \le c, \\ &y'(0) = \beta\, y(0), \\ &y'(c) = \beta\, y(c), \end{aligned}$$

has exactly one negative eigenvalue λ_0 and that this eigenvalue is independent on $c > 0$. Find λ_0 and an associated eigenfunction $y_0(x)$.
b) Determine whether or not $\lambda = 0$ is an eigenvalue. If yes, find an associated eigenfunction.