MATH 4567, FALL 2014
HOMEWORK PROBLEMS No.2
Due on March 3

Problem 1. On the interval $[-\pi, \pi]$ find the Fourier series
a) for the function f in Problem No. 1 on page 18;
b) for the function $f(x) = |x|$;
c) for the function $f(x) = \max\{x, 0\}$.

Problem 2. On the interval $[0, \pi]$ find
a) the Fourier sine series for the function f in Problem No. 2 (b) on page 12;
b) the Fourier cosine series for the function f in Problem No. 4 on page 13;
c) the Fourier cosine series for the function f in Problem No. 3 (a) on page 12.

In addition, in c) write down Parseval's equality corresponding to this Fouirer series and use it to evaluate

$$\sum_{n=1}^{\infty} \frac{1}{n^4}.$$

Problem 3. Find the best approximation g in the mean on the interval $0 \leq x \leq \pi$ for the function $f(x) = 1$ using linear combinations of

$$f_1(x) = \sqrt{\frac{2}{\pi}} \sin x, \quad f_2(x) = \sqrt{\frac{2}{\pi}} \sin 3x.$$

Then evaluate the error of approximation, that is, $\|f - g\|$ in $L^2[0, \pi]$.
Reminder: The set $\{f_1, f_2\}$ forms an orthonormal system in $L^2[0, \pi]$.

Problem 4. Let $S_n f(x)$ denote the n-th partial sum of the Fourier series in $-\pi \leq x \leq \pi$ for the function defined to be $f(x) = x + 1$ for $x > 0$, $f(x) = 2x - 3$ for $x < 0$, and $f(0) = 0$.

a) Evaluate for each $x \in [-\pi, \pi]$ the limit $S(x) = \lim_{n\to\infty} S_n f(x)$.
b) Sketch the graph of S on the whole real line.
c) Find the values $S(10)$, $S(20)$.

Hint: The function S is 2π-periodic, so it is enough to know its values on $(-\pi, \pi]$.

Problem 5. For the function $f(x) = \sqrt{1 - x^2}$ consider its Fourier series in the interval $-1 \leq x \leq 1$,

$$\sqrt{1 - x^2} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\pi n x) + b_n \sin(\pi n x).$$

Evaluate $\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$ without computing the Fourier coefficients a_n, b_n.
Hint: Apply Parseval's equality for Fourier series in $[-c, c]$. You will also need to compute a_0.