

Worksheet 2 $\frac{1}{2}$

Drawing surfaces is usually more difficult than drawing curves. Plotting just a few points on a surface rarely gives enough information about its shape. What we should do instead is plot several *curves* on the surface and then interpolate between them. To get these curves, we can use the method of cross-sections:

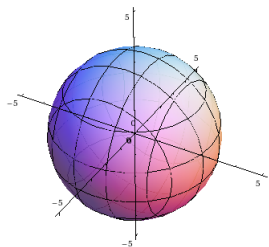
Step 1. Plug a 'good' value for one of the variables x , y or z into the given equation of a surface $f(x, y, z) = 0$. By doing it, we turn the original equation with three variables into a new (simpler) equation involving two variables only.

Step 2. Consider this new equation as an equation of the curve on the corresponding plane. Sketch this curve in the space.

Repeat these steps, if necessary.

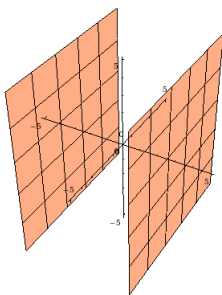
1. Sketch the surfaces given by the equations.

a) $x^2 + y^2 + z^2 = 10$



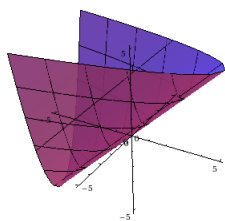
a sphere

c) $x^2 = 10$



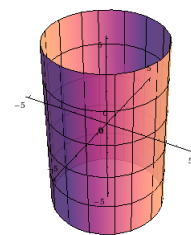
two parallel planes

e) $z = x + y^2$



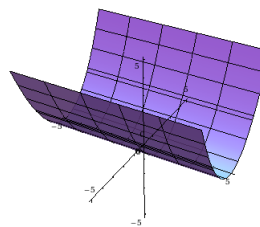
a parabolic cylinder

b) $x^2 + y^2 = 10$



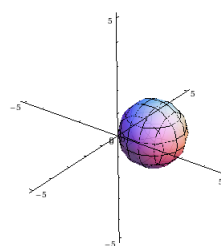
a cylinder

d) $y - \sqrt{3z} = 0$



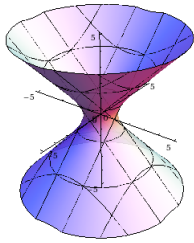
a parabolic cylinder

f) $x^2 + y^2 + z^2 - 2x - 2y = 0$



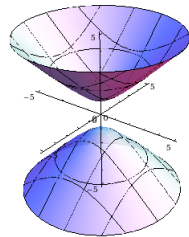
a sphere

g) $x^2 + y^2 - z^2 = 1$



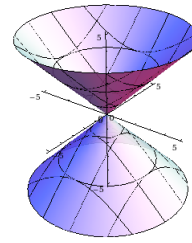
a hyperboloid of one sheet

i) $x^2 + y^2 - z^2 = -1$



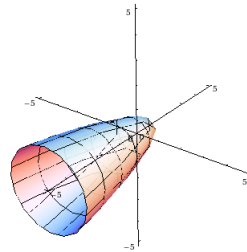
a hyperboloid of two sheets

h) $x^2 + y^2 - z^2 = 0$



a cone

j) $2x^2 + y + 2z^2 - 1 = 0$



a paraboloid

2. Let M be the point $(1, 1, 1)$. Does this point lie inside or outside of the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$? Does it lie above or below the paraboloid $x^2 + 2y^2 = 2z$?

Solution. A point $M(a, b, c)$ would lie inside the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ if

$$a^2 + 2b^2 + 3c^2 < 1.$$

In our case

$$1^2 + 2 \cdot 1^2 + 3 \cdot 1^2 > 1.$$

Hence, the point M is outside of the given ellipsoid.

To see if M lies above or below the given paraboloid, we plug $x = 1$, $y = 1$ into the equation $x^2 + 2y^2 = 2z$ and obtain

$$\begin{aligned} 1^2 + 2 \cdot 1^2 &= 2z \\ z &= \frac{3}{2}. \end{aligned}$$

So the point $(1, 1, \frac{3}{2})$ is on the paraboloid. But the z -coordinate of the point M is 1. Thus it lies *below* the paraboloid.

